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No. 884.

ON THE FLOW OF WATER OVER DAMS.

By GEORGE W. RAFTER, M. Am. Soc. C. E.

PRESENTED APRIL 18TH, 1900.

WITH DISCUSSION.

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Probably the main reason engineers have gone so far astray on this question has been the nearly entire lack of data applying to various

forms of crests. It is true that a few advanced hydraulicians have discussed the effect of variations in width of crest, but it has remained for H. Bazin, *Inspecteur Général des Ponts et Chaussées*, to elucidate the whole problem in a series of masterly studies, which may be found in *Annales des Ponts et Chaussées* for the years 1888, 1890, 1891, 1894, 1896 and 1898. These studies are, as regards detail and minute research, unparalleled.

Messrs. Fteley and Stearns,* have indeed shown the effect of width of crest on discharge over weirs. Their experiments, while decisive for the cases studied, are still quite limited in scope, but Bazin has determined coefficients for a very large number of cases, not only of crests of different widths, but with varying front and rear slopes, as well as for curved profiles. Indeed, taking into account the backward state of knowledge of flow over weirs, his work is in many respects revolutionary. Certainly, with the data now available, there is no excuse for using a sharp-crested weir formula in computations of flow over various-shaped crests, irregular or otherwise.

In making the foregoing comments, the writer has no intention of criticising the work of others. Indeed, he has himself groped blindly in the dark in this matter, as other engineers have done. His intention is to present an irrational practice so saliently that—with the data now at hand—from this time on, the use of sharp-crested weir formulas for computing flow over weirs of all sorts of shapes will be discontinued.

An illustration from the writer's experience is pertinent to the discussion. During the last twenty years he has had occasion to gauge streams extensively in various parts of the United States. Long experience convinced him several years ago that sharp-crested weir formulas were not applicable to dams with sloping crests, and accordingly, in arranging for an extensive series of gaugings of the Genesee River, over a sloping-faced dam at Mount Morris, New York, in 1893, he used the formula

$$Q = 3.4\,L\,H^{\frac{3}{2}}\dagger$$

as originally proposed for this dam by Augustus S. Kibbe, Jun. Am. Soc. C. E., in his report on Genesee River Storage, to John Bogart, M. Am. Soc. C. E., formerly State Engineer and Surveyor.

In 1896, after the Genesee River gaugings had been in progress at

* "Experiments on the Flow of Water, made during the Construction of Works for Conveying the Water of Sudbury River to Boston." By A. Fteley and F. P. Stearns, *Transactions*, Am. Soc. C. E., Vol. xii, p. 1.

† See Appendix F to Annual Report, State Engineer and Surveyor, for 1890, p. 455.

the dam of the Mount Morris Hydraulic Power Company for three years, a sharp-crested weir was erected on the Genesee River about 2½ miles above, where rock bottom clear across the river afforded an opportunity for such construction without heavy expense.*

TABLE No. 1.

H = head on weir, in feet.	h = head on dam, in feet.	Computed discharge over weir, in cubic feet per second, for heads = H.	Computed discharge over dam, in cubic feet per second, for heads = h.	Percentage differences in discharges.
(1)	(2)	(3)	(4)	(5)
0.50	0.60	185	135	—37.0
0.70	0.83	310	330	+ 6.0
0.80	0.90	450	445	− 1.0
1.02	1.00	540	505	− 7.0
1.86	1.55	1 325	1 260	− 5.0
2.01	1.75	1 490	1 605	+10.0
2.42	2.00	1 965	2 100	+ 7.0
2.65	2.50	2 250	3 230	+44.0.
3.20	2.75	2 990	3 860	+29.0
3.78	3.00	3 840	4 554	+19.0
4.37	3.25	4 770	5 280	+11.0
4.65†	3.35†	5 240	5 590	+ 7.0

† Approximate; taken from curve.

In order to correlate the measurements at the Hydraulic Power Company's dam with those at the weir, observations were taken at each place as nearly cotemporaneously as they could be made by a man going immediately from one to the other. Table No. 1 gives some of the heads actually observed at the weir and dam, together with the discharge over the weir in comparison with the computed discharge over the dam, and the percentage differences.

The crest of the Mount Morris Dam was quite irregular, and, in order to apply weir formulas, an accurate profile was taken and the crest sub-divided into a number of approximately level sections with each section computed separately, advancing by 0.1 ft. up to 10 ft. The flow over the entire dam was obtained by adding together the sums of the several sections at the corresponding heights, and tabulating them. The zero of the gauge was at the level of the lowest section.

The computed discharges, as shown by Columns (3) and (4), are somewhat irregular. This result is due to the disturbing effect of the irregular sections of the crest, the highest point of which was 2 ft. above the lowest.

* For detailed description of this weir see Appendix VII to Annual Report, State Engineer and Surveyor, for 1896, pp. 715–19.

Column (5) shows the percentage variations between the discharges as determined by a sharp-crested weir, up to 5 200 cu. ft. per second, and the discharges computed by the formula cited. These data show at once an error in judgment, excusable only because, previous to the publication of Bazin's paper of 1898, nobody knew how to do better.

In computing the Hudson River gaugings in 1895, the writer used the formula of General Mullins, as fairly applicable to a broad-crested dam like that at Mechanicsville, where Hudson River gaugings have been kept continuously from October, 1887.*

In August, 1898, the writer began an extensive special investigation as to water supply for summit-level canals in the State of New York, for the United States Board of Engineers on Deep Waterways. The magnitude of the commercial interests involved justified a most thorough study, and the work was accordingly carried out on an extended scale. A large collection of new data has been obtained, which, by permission of the Board, the writer has the pleasure of presenting to the Society herein.

At the beginning of the study it was deemed advisable to gauge a number of streams tributary to proposed deep waterways in Central and Eastern New York, not only with reference to extending information as to the low-water flow of the Oswego, Mohawk, Black and Hudson Rivers, but especially to gain more definite information as to the flood flows of these streams and their tributaries, it being recognized clearly that the control of floods in canalized river-beds was a serious feature of the general problem.

To accomplish this, gauging stations were established on Seneca River, at Baldwinsville; Oswego River, at Fulton; Chittenango Creek, at Bridgeport; Oneida Creek, at Kenwood; West Branch of Fish Creek, at McConnellsville; East Branch of Fish Creek, above Point Rock; Salmon River, above High Falls; Mohawk River, at Ridge Mills, Little Falls and Rexford Flats; Nine Mile Creek, below Stittville; Oriskany Creek, at Oriskany and Coleman; Saquoit Creek, at New York Mills; West Canada Creek, at Dolgeville; Garoga Creek, 3 miles above mouth; Cayadutta Creek, below Johnstown, and Schoharie Creek, at Fort Hunter. In addition, gaugings of Hudson River, at Mechanicsville and Fort Edward, and of Schroon River, at Warrensburg,

*For this formula see Mullins' Irrigation Manual, pp. 11-12, 138-139, 171-172. Also see Annual Report, State Engineer and Surveyor, of New York, for 1895, pp. 105-106; and Water Supply and Irrigation Papers of United States Geological Survey, No. 24; Water Resources the of State of New York, Part I, pp. 79-80.

at stations established previously by the writer, were available, as well as of Black River, at Huntingtonville, a suburb of Watertown, at a station established by the Board of Water Commissioners of Watertown.

The foregoing gauging stations are in every case existing dams, either of masonry or timber. Several of them, as at Baldwinsville, Fulton, Little Falls, Middleville, Dolgeville, etc., have extensive power devolopments, with large quantities of water passing through turbine water wheels, for either the whole or a portion of each day. Hardly any two cross-sections are alike, as may be sufficiently appreciated by examining Figs. 8 to 20, although some of them conform generally to certain of Bazin's types, as is shown by the illustrations. Finally, many of them have gross irregularities in the crests, longitudinally, as shown. The method of treatment, in order to obtain approximately correct results, becomes, therefore, a matter of some difficulty. In a few cases, as on Nine Mile Creek, West Canada Creek, etc., where the crests were very irregular, a small amount of work was done in the way of leveling them. Generally, however, the crests were left nearly in the same condition as found, a profile was carefully taken and the crest divided into a series of approximately level sections for computation. A gauging blank was furnished the gauge readers, with columns for entering depth on crest of dam, A. M. and P. M., number of water wheels used, size of same, name of manufacturer and daily run, working head on wheels, readings of head-race and tail-race gauges, and other information necessary for keeping an accurate account of the water passing over the crest in 24 hours, as well as through water wheels for the same period. Gauge readers were employed to take these readings twice each day.

In order to obtain flows through water wheels, recourse was had to records of the test flume of the Holyoke Water Power Company, of Holyoke, Mass., where the principal wheels now in common use in New York State have, at one time or another, been tested. On requesting a record of such tests, as applying to wheels at the several gauging stations, the Holyoke Water Power Company kindly responded that they would furnish the records under the condition that they be not published unless the consent of parties for whom the wheels had been tested were first obtained. This condition being assented to, information was furnished as to tests of the principal wheels in use, giving proportional part of opening of speed gate for various conditions of tests, revolutions

of wheel, quantity of water discharged, power developed, efficiency, etc. From these records, wheel-discharge curves have been prepared for the water wheels in use at each dam. By the use of such curves, derived from actual tests, it is believed that the discharges through turbine water wheels at the various gauging stations have been computed with a very high degree of accuracy. Under these conditions turbine water wheels become in effect efficient water meters. In a few cases, where there were no tests applying, the discharges as per manufacturers' tables have been used. The writer's thanks are due to the Holyoke Water Power Company for the courtesy of furnishing these useful data.

Before describing the method of procedure for obtaining flows over dams at the several gauging stations, we may refer briefly to some of the more salient points of Bazin's papers in *Annales des Ponts et Chaussées.*

In the beginning of his first paper, Bazin remarks that the theory of the weir is the least advanced of all branches of hydraulics. The coefficients used in practice vary between such wide limits that in most cases we are unable to make a rational selection from the many numerical values assigned to them.

The problem, he says, is in fact a complicated one, being connected on the one hand with the theory of flow through orifices and on the other with that of open channels. The value of the coefficients in each case is influenced by many elements. Thus we ought to consider:

(1) The velocity of approach; that is, the velocity with which the up-stream water reaches the weir, the effect of which cannot be neglected in weirs of small height.

(2) The contraction of the vertical section of the stream at the weir, the amount depending upon the height of the weir and the form of the crest.

(3) The lateral contraction which, though unimportant in weirs of great length, seriously modifies the results in shorter weirs.

As a further condition, Bazin points out that when the down-stream channel has a width of the length of the weir, so that the overflowing sheet of water, or nappe, touches at the sides, thus preventing free admission of air under the nappe, there occur special phenomena greatly affecting the flow.*

* Bazin's earlier papers are directed specially to a detailed investigation of these several points. Space will not be taken here to describe his experiments in detail. The original data may be found in the *Annales des Ponts et Chaussées* for the years already cited. A translation of the earlier numbers has also been made by Messrs. Arthur Marichal and John C. Trautwine. Jr., and may be found in the *Proceedings* of the Engineers' Club of Philadelphia for January, 1890; July, 1892; October, 1892, and April, 1893.

Bazin's method of experimentation may be referred to briefly. A standard weir was set up at the head of a long chamber, in which the actual volume passing over was measured a sufficient number of times to give averages, which Bazin considers are accurate to within probably less than 1 per cent. Having established in this way the values of the coefficients for a standard weir, with heads varying from about 0.164 ft. to 1.969 ft., the experiments on weirs of irregular profiles were made by placing each experimental weir below the standard weir, and observing the heads synchronously on each. In these experiments a steady current was established in the channel, and observations of the known volume passing over the standard weir were made, which volume also passed over the weir under investigation, lower down.

If we let H and h denote, respectively, the head upon the standard weir and upon the lower weir, L and l, their corresponding lengths, and M and m, the coefficients of discharge, and then, adopting provisionally Formula (1) for the standard weir—

$$Q = M L H \sqrt{2 g H}; \qquad \qquad (1)$$

and similarly for the lower weir

$$Q = m l h \sqrt{2 g h}. \qquad \qquad (2)$$

Equating these two values of Q, we have

$$M L H \sqrt{H} \sqrt{2g} = m l h \sqrt{h} \sqrt{2g}, \text{ or}$$

$$M L H^{\frac{3}{2}} = m l h^{\frac{3}{2}}$$

from which we deduce the value of m:

$$m = M \left(\frac{L}{l}\right) \times \left(\frac{H}{h}\right)^{\frac{3}{2}}$$

or, conversely: $$M = m \left(\frac{l}{L}\right) \times \left(\frac{h}{H}\right)^{\frac{3}{2}}$$

As already stated, Bazin's preliminary gauging operations gave, once for all, the coefficient M for the standard weir for each value of H. The ratio $\frac{L}{l}$, which is very nearly unity, remained constant for all experiments of any one series, and, therefore, we have only to measure the heads H and h in order to obtain the coefficient m.

Fteley and Stearns experimented somewhat on the influence of the height of the weir upon the flow, and probably as interesting a point as any brought out by Bazin's extended discussion is the considerable influence of this element upon the flow. After presenting the detail

TABLE No. 2.—VALUES OF THE COEFFICIENT m IN THE FORMULA $Q = m l h \sqrt{2 g h}$, FOR A SHARP-CRESTED WEIR WITHOUT LATERAL CONTRACTION, THE AIR BEING ADMITTED FREELY BENEATH THE OVERFLOWING SHEET OF NAPPE.

Observed head = h, in feet.	Values of the coefficient m corresponding to the height p of the weir above the bottom of the channel.									Limiting value of m = coefficient n.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Value of p, in feet.	0.656	0.984	1.312	1.640	1.968	2.624	3.280	4.920	6.560	∞
0.164	0.458	0.453	0.451	0.450	0.449	0.449	0.449	0.448	0.448	0.4481
0.197	0.456	0.450	0.447	0.445	0.445	0.444	0.443	0.443	0.443	0.4427
0.230	0.455	0.448	0.445	0.443	0.442	0.441	0.440	0.440	0.439	0.4391
0.262	0.456	0.447	0.443	0.441	0.440	0.438	0.438	0.437	0.437	0.4363
0.295	0.457	0.447	0.442	0.440	0.438	0.436	0.436	0.435	0.434	0.4340
Means..	0.456	0.449	0.446	0.444	0.443	0.442	0.441	0.441	0.440	0.4400
$m\sqrt{2g}$	3.66	3.60	3.58	3.56	3.55	3.54	3.54	3.53	3.53	3.53
0.328	0.459	0.447	0.442	0.439	0.437	0.435	0.434	0.433	0.433	0.4322
0.394	0.462	0.448	0.442	0.438	0.436	0.433	0.432	0.430	0.430	0.4291
0.459	0.466	0.450	0.443	0.438	0.435	0.432	0.430	0.428	0.428	0.4267
0.525	0.471	0.453	0 444	0.438	0.435	0.431	0.429	0.427	0.426	0.4246
0.591	0.475	0.456	0.445	0.439	0.435	0.431	0.428	0.426	0.425	0.4229
Means..	0.467	0.451	0.443	0.438	0.436	0 432	0.431	0.429	0.428	0.4271
$m\sqrt{2g}$	3.74	3.62	3.56	3.52	3.50	3.47	3.46	3.44	3.44	3.43
0.656	0.480	0.459	0.447	0.440	0.436	0.431	0.428	0.425	0.423	0.4215
0.722	0.484	0.462	0.449	0.442	0.437	0.431	0.428	0.424	0.423	0.4203
0.787	0.488	0.465	0.452	0.444	0.438	0.432	0.428	0.424	0.422	0.4194
0.853	0.492	0.468	0.455	0.446	0.440	0.432	0.429	0.424	0.422	0.4187
0.919	0.496	0.472	0.457	0.448	0.441	0.433	0.429	0.424	0.422	0.4181
Means..	0.488	0.465	0.452	0.444	0.438	0.432	0.428	0.424	0.422	0.4196
$m\sqrt{2g}$	3.92	3.73	3.63	3.56	3.52	3.46	3.44	3.40	3;39	3.37
0.984	0.500	0.475	0.460	0.450	0.443	0.434	0.430	0.424	0.421	0.4147
1.050	0.500	0.478	0.462	0.452	0.444	0.436	0.430	0.424	0.421	0.4168
1.116	0.500	0.481	0.464	0.454	0.446	0.437	0.431	0.424	0.421	0.4162
1.181	0.500	0.483	0.467	0.456	0.448	0.438	0.432	0.424	0.421	0.4156
1.247	0.500	0.486	0.469	0.458	0.449	0.439	0.432	0.424	0.421	0.4150
Means..	0.500	0.481	0.464	0.454	0.446	0.437	0.431	0.424	0.421	0.4162
$m\sqrt{2g}$	4.01	3.86	3.73	3.64	3.58	3.50	3.46	3.40	3.38	3.34
1.312	0.500	0.489	0.472	0.459	0.451	0.440	0.433	0.424	0.421	0.4144
1.378	0.500	0.491	0.474	0.461	0.452	0.441	0.434	0.425	0.421	0.4139
1.444	0.500	0.494	0.476	0.463	0.454	0.442	0.435	0.425	0.421	0.4134
1.509	0.500	0.496	0.478	0.465	0.456	0.443	0.435	0.425	0.421	0.4128
1.575	0.500	0.496	0.480	0.467	0.457	0.444	0.436	0.425	0.421	0.4122
Means..	0.500	0.493	0.476	0.463	0.454	0.442	0.435	0.425	0.421	0.4133
$m\sqrt{2g}$	4.01	3.96	3.82	3.72	3.64	3.55	3.49	3.41	3.38	3.32
1.640	0.500	0.496	0.482	0.468	0.459	0.445	0.437	0.426	0.421	0.4118
1.706	0.500	0.496	0.483	0.470	0.460	0.446	0.438	0.426	0.421	0.4112
1.772	0.500	0.496	0.485	0.472	0.461	0.447	0.438	0.426	0.421	0.4107
1.837	0.500	0.496	0.487	0.473	0.463	0.448	0.439	0.427	0.421	0.4101
1.903	0.500	0.496	0.489	0.475	0.464	0.449	0.440	0.427	0.421	0.4096
1.969	0.500	0.496	0.490	0.476	0.466	0.451	0.441	0.427	0.421	0.4092
Means..	0.500	0.496	0.486	0.472	0.462	0.448	0.436	0.427	0.421	0.4104
$m\sqrt{2g}$	4.01	3.98	3.90	3.79	3.71	3.60	3.50	3.42	3.38	3.29

of experiments on sharp-crested weirs of various heights and for various heads between the limits stated, Bazin gives a table of values of the coefficient m for sharp-crested weirs, ranging in height from 0.656 ft. to 6.56 ft. (0.2 to 2.0 m.). Column (11) of Table No. 2 gives the limiting value of m, which equals the coefficient μ of Bazin's formulas, (represented by n) and which represents the value of m for a weir of infinite height, or of such a height that the height of the weir above the bottom of the channel has no further effect upon the flow. As will be seen by examining this table, the influence beyond 6.56 ft. is only slight.

The formula for a sharp-crested weir without end contractions, as ordinarily used in the United States, is that of Mr. Francis, namely:

$$Q = ClH^{\frac{3}{2}}.$$

In this formula, it is necessary to take into account the velocity of approach. In the formula as it here stands, we may consider h as the uncorrected form, while we may consider that in H all corrections are made. Usually, in practice, such corrections are conveniently made by adding the correction factor to the formula, a simple form of tabulation being used.

In Bazin's formula it is equally necessary to take into account the velocity of approach, although his coefficient. m, apparently includes that element within itself. This is, however, for convenience merely, the more especially because within the range of his experiments the velocity of approach was mostly a relatively unimportant element. Nevertheless, Bazin gives elaborate corrections for its use, when necessary. It is, however, very important to understand that, while Bazin's experiments do not include a separate correction for velocity of approach, but that its effect is included in m, nevertheless, when, as in the present case, very high heads are involved, the correction can be more conveniently made by a separate correction factor than in any other manner, and accordingly this is the method which has been applied in the Cornell University experiments following. These coefficients are, therefore, corrected for velocity of approach, and are ready to be used without further corrections of any kind. The values of $m\sqrt{2g}$ for different heads on the crest and for heights of weir varying from 0.656 ft. to 6.56 ft., and also for the limiting value of $m = n$ may be seen as carried out in Table No. 2.

In regard to the accuracy of the coefficients given by this table, Bazin remarks that, except in the unusual case of a very low weir, which should always be avoided, it will give the coefficient m within 1%, provided, however, that the arrangements of his standard weir be exactly reproduced. It is also pointed out as especially important that the admission of air behind the falling sheet be perfectly assured; otherwise m may vary within much wider limits.

Description of Bazin's Experiments on Weirs of Irregular Cross-Section.

The following statements have been condensed from Bazin's paper in *Annales des Ponts et Chaussées*, for 1898.

We will now consider weirs in which the back and front faces, instead of being vertical, have a slope of greater or less inclination. The conditions of discharge will be found to be greatly modified. The slope on the up-stream side tends to diminish the contraction in passing over the crest, and hence to increase the discharge. The influence of the down-stream slope, on the other hand, is not always constant, but varies according to the degree of inclination. If the down-stream face is not far from vertical, the nappe adheres to it for small discharges, but becomes detached at a certain head and then follows the condition described as wetted underneath, analogous to that which we have studied for square timber weirs. If, on the contrary, the slope is nearly horizontal, the nappe does not detach itself, but remains in contact with the face of the weir under all heads. The discharge may, however, vary greatly according to the degree of inclination of the face. On the other hand, the influence of the width of crest is considerable, as we have seen in the case of weirs formed of square timbers. Hence, a weir with a wide crest and inclined faces may present a large variety of results, each type having, so to speak, its own special scale of coefficients. Such a study, to be complete, must include a very considerable number of particular cases. Without embracing all possible cases, the experiments made have been somewhat numerous. They include weirs with different degrees of slope on the front and back faces, and having sharp crests, on the one hand, and on the other, crests 0.10, 0.20 and 0.40 m. in width (0.328, 0.656 and 1.312 ft., respectively).

Some additional experiments have been made on weirs having crests joined to the inclined faces by curved surfaces, and, finally, weirs having completely curved profiles have been experimented upon.

We may divide the numerous series of experiments into five groups, namely:

(1) Weirs so nearly vertical on the down-stream side that the nappe remains detached.

(2) Weirs having the up-stream face vertical, or nearly vertical, but with a slope on the down-stream face so nearly horizontal that the water always remains in contact.

(3) Weirs both faces of which are at an inclination differing from the horizontal by less than 45 degrees.

(4) Weirs in which the crest is joined to the inclined faces by curved surfaces.

(5) Weirs having completely curved profiles.

First Group.—Weirs Having the Down-Stream Face Vertical or Nearly Vertical.

This group includes ten series of experiments, the results of which differ notably, according to inclination of slope and width of crest. The values of the coefficient $\frac{m}{m'}$,* which have been obtained for sharp-crested weirs, have been compared with those corresponding to nappes wetted underneath with sharp-crested weirs. Similarly, we may compare conveniently the values obtained for weirs with crests 0.10 and 0.20 m. wide (0.328 and 0.656 ft., respectively), with those for flat-crested beam weirs of the same width in the table.†

The coefficients have been made to follow a uniform law, by plotting the immediate results of the experiments in such a manner that the head h represents the abscissa and the ratio $\frac{m}{m'}$ the ordinate of a point representing one of the experiments. A mean curve has been drawn by the aid of these points, and from this curve, drawn on a large scale, values of the ratio $\frac{m}{m'}$ have been taken, corresponding, roundly, to abscissas of 0.10 m., 0.15 m., etc. (0.328 ft., 0.492 ft., etc.). All the ex-

* This may be defined as the ratio of the coefficient of the weir under special consideration to that of the standard weir of comparison.

† See p. 159, *Annales des Ponts et Chaussées, 2d Trimestre*; 1898.

periments terminate with nappes wetted underneath, but the discharge for these, as well as for depressed and adhering nappes, is shown in the table, the nature of the nappe in each case being indicated in the proper column. If we consider, first, sharp-crested weirs, we perceive that the appearance of the wetted nappe is preceded by the depressed nappe, imprisoning air between its under surface and the body of the weir, excepting in the case of a weir with a face batter of 3 : 2, which permits the formation of an adhering nappe. When once the wetted nappe is established, its coefficient does not differ greatly from that for a sharp-crested weir, excepting in the cases where the slope on the back is 3 : 1 and 3 : 2, when it is greater. For weirs with flat crests 0.10 m. (0.328 ft.) wide, the adhering form of nappe appears when the up-stream face has a slope of 3 : 1 or 3 : 2. The ratio $\frac{m}{m'}$ is also modified, however, by adherence of the water to the flat crest. At the moment when this adhesion ceases, the coefficient diminishes suddenly about 10 per cent. This has taken place in the two series, Nos. 133 and 134, where the up-stream face is vertical. In the other series, the head has not been sufficiently large to detach the nappe, and the coefficient remains, in these experiments, greater in value than it would be for a beam weir in which the nappe has become detached before the given head is attained.

Only one series of experiments has been made on weirs with crest 0.20 m. (0.656 ft.) wide, the down-stream face being vertical and that on the up-stream side having a slope of 1 : 2. In accordance with the increased width of crest the wetted nappe does not appear until quite late, and with a coefficient notably diminished. It is clear that more extended experiments would have led in all cases to results differing in a similar manner from those for crests 0.10 m. in width. Without going farther we may say that, on a weir having the down-stream face nearly vertical, the wetted form of nappe always appears when a certain head has been attained. This limiting head varies with the inclination of the front and back faces, and is never the same as that which corresponds to detachment of the nappe from the flat crest, which also influences the value of the coefficient. Each type of weir requires a special study, and, in view of the complexity of conditions involved, it is impossible to establish a general formula for the discharge coefficient.

Second Group.—Face of the Weir on the Up-Stream Side Vertical or Nearly Vertical.

Let us now consider the case where, in contradistinction to the first group, the back is nearly vertical and the down-stream face has considerable slope. For sharp-crested weirs with the up-stream face vertical the ratio $\frac{m}{m'}$ is nearly constant for each series, the mean values being 1.13, 1.03, 0.90 and 0.84 for slopes on the down-stream face of 1 : 1, 1 : 2, 1 : 5 and 1 : 10, respectively. Where the up-stream face has a slope of 3 : 1 or 2 : 1, it produces an increase, in the value of the coefficient, of a few per cent. Turning to the experiments on weirs with crests 0.10, 0.20 and 0.40 m. (0.328, 0.656 and 1.312 ft., respectively) in width, it may be seen that the coefficients increase with the head, Series No. 143 only showing, for the final values, a rapid diminution, proving that the nappe has become detached. It is necessary, in fact, in order that the nappe shall become detached, that the head shall be greater in proportion as the width of the crest becomes greater and the slope of the down-stream face more gentle. This limit has not, in general, been reached in the experiments, and the nappes remain attached to the crests, excepting for Series No. 143 (slope on the down-stream side 1 : 1), where the detachment of the nappe leads to a diminution of the coefficient ratio from 1.21 to 1.14. The next series, No. 144, indicates also a slight tendency of the ratio $\frac{m}{m'}$ to decrease, the slope here being 1 : 2. This tendency entirely disappears where the slope of the down-stream face is not greater than 1 : 3 or 1 : 4. Omitting the results for the relatively small heads of 0.10 m. (0.328 ft.) or less, where there are some irregularities, the increments of the coefficient ratio $\frac{m}{m'}$ for heads from 0.10 m. (0.328 ft.) to the limits of the experiments, are given for each series, in comparison with the results for a weir with a crest 0.10 m. (0.328 ft.) in width, Series Nos. 133 and 134, and with those for flat-crested weirs, 0.40 m. (1.312 ft.) and 2.00 m. (6.56 ft.) in width, Series Nos. 113 and 115. Such comparison shows, first, that for the same width of crest, $\frac{m}{m'}$ diminishes when the inclination of the down-stream face is gradually diminished below 45°; second, that, other things being equal, that is to say, for the same slope on the two faces in each case, the ratio $\frac{m}{m'}$ diminishes when the width of crest is increased.

Third Group.—Slope of the Up-Stream and Down-Stream Faces Very Gentle, not Exceeding 45 Degrees.

Weirs encountered in practice do not often have nearly vertical faces like those we have been considering, but have slopes inclined 45°, or more, from the vertical. Such weirs have been made the subject of a series of experiments, in which slopes of 1 : 1 and 1 : 2 on the up-stream side have been combined with slopes of 1 : 1, 1 : 2 and 1 : 5 on the down-stream side for three different widths of crest.

In most cases the ratio $\frac{m}{m'}$ increases with the head, but, in order to investigate more fully the changes in value of $\frac{m}{m'}$, it is necessary to consider separately the case of sharp-crested weirs as distinguished from those with wide crests.

Sharp-Crested Weirs.

Slope 1 : 1 on Down-Stream Side.—The coefficient ratio $\frac{m}{m}$, decreases as the head h increases from above 1.20 for very slight heads to 1.11 or 1.12 for the greatest heads used. Its value is sensibly the same for the two slopes of 1 : 1 and 1 : 2 on the back. The rate of decrease is not uniform, being very gradual to $h = 0.30$ m. (0.984 ft.), beyond which it changes rapidly, without doubt due to the detachment of the nappe.

Slope 1 : 2 on Down-Stream Side.—Instead of decreasing as the head increases, $\frac{m}{m'}$, increases slowly in value between the limits 1.10 and 1.13, its value being nearly the same for the two slopes of 1 : 1 and 1 : 2 on the back.

Slope 1 : 5 on Down-Stream Side.—In this case the coefficient ratio is nearly independent of h, decreasing from 1.015 to 1.000 for a slope of 1 : 1 on the back, and from 1.045 to 1.035 for a slope of 1 : 2 on the back.

Weirs Having Crests 0.10 and 0.20 M. (0.328 and 0.656 Ft.) in Width.

The coefficient always increases with the head, but the limits between which this increase takes place differ in each case. If it were possible to increase the head indefinitely, and at the same time the height of the weir, the conditions of discharge would approach progressively those for a sharp-crested weir, the width of the crest

becoming more and more negligible, relative to the general dimensions of the dam. The series of coefficients relating to crests of a given width cannot be, with certainty, extended beyond the experimental limits between which they were obtained. If, however, the curves representing the coefficients were prolonged sufficiently, they would converge toward those which correspond to a sharp-crested weir with vertical faces. The slope on the back determines the direction of the filaments which constitute the inferior surface of the nappe, and influences thus the contraction at the inner edge of the crest, and hence also the discharge. The slope on the down-stream side, on the other hand, affects the pressure underneath the nappe.

When width of crest is not negligible, the inclination of the down-stream face determines the limiting head at which the nappe detaches itself from the crest, and at which point the conditions of discharge change suddenly. This limit depends also, in a certain measure, on the velocity of approach, or, which amounts to the same thing, on the ratio of the head h to the height p of the weir. A complete formula should include, in addition to the slopes of the two faces, the two ratios, $\frac{h}{c}$ and $\frac{h}{p}$, where c is the width of the flat portion of the crest. Such a formula would be excessively complicated.

Fourth Group.—The Two Faces United by a Curved Surface at the Crest of the Weir.

Seven types of weirs were experimented on, with a view to determining the effect on the discharge of joining the two faces of the weir by a curved surface, the slopes used being, on the up-stream side, very nearly vertical ($\frac{1}{5}$: 1), and on the down-stream side from 3 : 1 to 5 : 1. The up-stream edge of the crest being rounded to an arc of 0.05, 0.10 or 0.20 m. (0.164, 0.328 or 0.656 ft.) radius.

Types Nos. 1 and 2 differ only in regard to the radius of curvature of the back edge of the crest, this being 0.05 m. (0.164 ft.) for the first, and 0.10 m. (0.328 ft.) for the second. The radius of 0.10 m. increases the discharge slightly more than that of 0.05 m., though the difference is unimportant, but the values of $\frac{m}{m'}$ in both cases surpass considerably those which have been obtained for similar weirs with the two

faces united by flat crests 0.10 and 0.20 m. (0.328 and 0.656 ft.) in width. Series Nos. 145 and 153.

Types Nos. 3 and 4 are similar to Types Nos. 1 and 2, except as regards the inclination on the down-stream side, which has been reduced from 1 : 3 to 1 : 5. This conduces to equalize the values of *m*, which differ relatively little from each other, although the values are sensibly less at the same heads than those for Types Nos. 1 and 2, and yet somewhat greater than for weirs with flat crests, 0.10 and 0.20 m. (0.328 and 0.656 ft.) in width. Series Nos. 145, 155 and 156.

Types Nos. 5 and 6 differ only by the radius of curvature at the back, which is 0.10 and 0.20 m., respectively, the crest being wider than in the preceding cases. The value of *m* is somewhat less than before, not differing greatly from that which corresponds to Types Nos. 3 and 4.

The width of crest is increased still further in Type No. 7, the length of the rectilinear section between the origins of the two curved surfaces being 0.20 m. (0.656 ft.). This modification produces a sensible diminution in the value of *m*.

Fifth Group.—Weirs with Completely Curved Profiles.

We turn, finally, to the consideration of weirs having completely curved profiles. The coefficient *m* then attains exceptionally high values. Types Nos. 1 and 2, having vertical down-stream faces, permit of the formation of nappes wetted underneath. The corresponding coefficients are much higher than for the analogous case with a sharp-crested weir.*

Types Nos. 3 and 4 have crests which differ only in the radii of curvature of the curved surfaces, being 0.05 and 0.08 m. (0.164 and 0.2624 ft.), respectively, for Type No. 3, and 0.10 and 0.12 m. (0.328 and 0.3936 ft.), respectively, for Type No. 4. The crest, in this latter case, is much larger and the coefficient *m* is, as a result, notably less for heads up to 0.30 m. (0.984 ft.), but it is not the same beyond this point, for the concave form of Type No. 3 tends to produce detachment of the nappe, and the coefficient for this type continues to diminish from this point, becoming less than for Type No. 4, in which the coefficient increases for all heads within the limits of the experiments.

* For the type-forms under this group, see Bazin's paper.

The results of Bazin's experiments on weirs of irregular profiles, except a few of the series which have been omitted, will be found on the pages indicated in the following list. The value of $m \sqrt{2g}$ is given in every case as derived from Bazin's tabulated value of m:

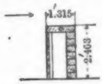
BAZIN'S EXPERIMENTS.

Series No.	Page.	Series No.	Page.	Series No.	Page.
113	237	140	241	163	248
114	275*	141	242	164	248
115	277*	142	242	165	248
116	276*	143	242	166	249
117	278*	144	243	167	249
125	237	145	243	168	249
126	237	146	243	170	250
127	238	147	244	172	250
128	238	149	244	173	274*
129	238	150	244	175	250
130	267*	151	245	176	251
131	239	153	245	177	251
132	239	154	245	178	273*
133	239	156	246	179	251
134	240	157	246	181	252
135	268*	158	246	182	252
136	240	159	247	188	252
137	240	160	247	189	253
138	241	161	247	193	253
139	241	162	272*		

* With diagram of similar Cornell experiment.

BAZIN'S

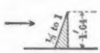
SERIES NO. 113



No. of Experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$	
1	0.208	0.3294	2.64	
2	0.289	0.3313	2.66	
3	0.363	0.3307	2.66	
4	0.443	0.3302	2.65	
5	0.518	0.3321	2.66	
6	0.592	0.3348	2.64	
7	0.667	0.3378	2.71	
8	0.736	0.3427	2.75	
9	0.805	0.3468	2.78	
10	0.863	0.3494	2.80	
11	0.936	0.3547	2.85	
12	0.989	0.3593	2.88	
13	1.035	0.3628	2.91	
14	1.076	0.3660	2.94	
15	1.114	0.3682	2.95	
16	1.159	0.3734	3.00	
17	1.197	0.3758	3.01	
18	1.252	0.3821	3.06	
19	1.320	0.3877	3.11	

BAZIN'S

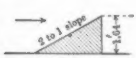
SERIES NO. 125



1	0.318	0.4516	3.62	— Nappe depressed.
2	0.383	0.4530	3.63	
3	0.439	0.4604	3.69	
4	0.492	0.4715	3.79	
5	0.532	0.4978	4.00	
6	0.569	0.5258	4.22	— Nappe wetted underneath.
7	0.633	0.5178	4.16	
8	0.691	0.5187	4.16	
9	0.752	0.5100	4.09	
10	0.818	0.5082	4.09	
11	0.878	0.5047	4.05	
12	0.950	0.4896	4.00	
13	1.006	0.4996	4.01	
14	1.077	0.4954	3.97	
15	1.140	0.4905	3.93	
16	1.190	0.4919	3.95	
17	1.266	0.4892	3.92	
18	1.328	0.4883	3.91	
19	1.395	0.4863	3.90	

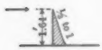
BAZIN'S

SERIES NO. 126



1	0.295	0.5015	4.02	— Nappe depressed.
2	0.360	0.5028	4.03	
3	0.413	0.5024	4.03	
4	0.467	0.5138	4.12	
5	0.520	0.5205	4.17	
6	0.567	0.5332	4.27	— Nappe wetted underneath.
7	0.627	0.5256	4.21	
8	0.684	0.5238	4.20	
9	0.740	0.5251	4.21	
10	0.804	0.5244	4.20	
11	0.860	0.5234	4.20	
12	0.920	0.5233	4.20	
13	0.981	0.5203	4.17	
14	1.040	0.5228	4.19	
15	1.105	0.5175	4.15	
16	1.164	0.5197	4.17	
17	1.222	0.5170	4.15	
18	1.284	0.5187	4.16	
19	1.351	0.5142	4.12	

BAZIN'S
SERIES NO. 127.*



No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$	
1	0.324	0.4326	3.47	Nappe depressed
2	0.389	0.4384	3.51	
3	0.449	0.4419	3.54	
4	0.509	0.4454	3.57	
5	0.561	0.4606	3.69	
6	0.608	0.4728	3.80	
7	0.653	0.4933	3.95	
8	0.697	0.5072	4.07	Nappe wetted
9	0.764	0.4989	4.00	underneath.
10	0.831	0.4935	3.95	
11	0.896	0.4895	3.93	
12	0.961	0.4885	3.92	
13	1.033	0.4812	3.86	
14	1.093	0.4823	3.87	
15	1.166	0.4720	3.79*	
16	1.228	0.4746	3.81	
17	1.292	0.4729	3.80	
18	1.360	0.4726	3.79	
19	1.424	0.4715	3.78	

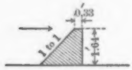
*See Bazin's observations relative to the behavior of the nappe for Series Nos. 127 and 128.

BAZIN'S
SERIES NO. 128.*

1	0.295	0.5015	4.02	Nappe adhering.
2	0.353	0.5104	4.09	
3	0.407	0.5090	4.09	
4	0.464	0.5107	4.10	
5	0.526	0.5068	4.07	
6	0.580	0.5159	4.14	
7	0.626	0.5183	4.16	
8	0.692	0.5179	4.15	
9	0.737	0.5156	4.14	
10	0.805	0.5178	4.15	
11	0.843	0.5133	4.12	
12	0.890	0.4909	3.94	Nappe wetted
13	0.925	0.4955	3.97	underneath.
14	0.967	0.4849	3.84	
15	1.023	0.4834	3.87	
16	1.096	0.4820	3.87	
17	1.166	0.4766	3.83	
18	1.230	0.4758	3.82	
19	1.294	0.4725	3.79	
20	1.360	0.4733	3.80	
21	1.430	0.4696	3.77	

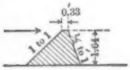
*See Bazin's observations relative to the behavior of the nappe for Series Nos. 127 and 128.

BAZIN'S
SERIES NO. 129.



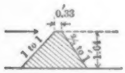
1	0.343	0.4064	3.25	Nappe depressed
2	0.399	0.4225	3.39	
3	0.456	0.4343	3.48	
4	0.511	0.4463	3.57	
5	0.565	0.4577	3.67	
6	0.612	0.4722	3.79	Nappe wetted
7	0.665	0.4778	3.83	underneath and
8	0.726	0.4838	3.88	attached to flat
9	0.774	0.4952	3.97	crest.
10	0.830	0.4982	4.00	
11	0.885	0.5029	4.04	
12	0.943	0.5079	4.08	
13	0.995	0.5116	4.11	
14	1.047	0.5155	4.14	
15	1.102	0.5187	4.16	
16	1.163	0.5174	4.15	
17	1.212	0.5236	4.21	
18	1.266	0.5263	4.23	
19	1.327	0.5276	4.23	

BAZIN'S
SERIES NO. 131



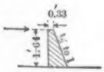
No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$	
1	0.347	0.3989	3.20	Nappe depressed.
2	0.336	0.4140	3.31	Nappe adhering.
3	0.395	0.4298	3.45	
4	0.450	0.4326	3.47	
5	0.504	0.4443	3.56	
6	0.564	0.4611	3.70	
7	0.610	0.4755	3.82	
8	0.667	0.4770	3.83	
9	0.720	0.4882	3.91	
10	0.773	0.4937	3.96	Nappe wetted underneath and attached to flat crest.
11	0.835	0.4948	3.97	
12	0.889	0.4997	4.01	
13	0.943	0.5048	4.05	
14	0.998	0.5090	4.08	
15	1.046	0.5128	4.12	
16	1.098	0.5172	4.15	
17	1.161	0.5167	4.15	
18	1.213	0.5226	4.20	
19	1.264	0.5249	4.21	
20	1.333	0.5242	4.21	
21	1.394	0.5234	4.20	

BAZIN'S
SERIES NO. 132



No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$	
1	0.337	0.4128	3.31	Nappe adhering.
2	0.398	0.4251	3.41	
3	0.453	0.4344	3.48	
4	0.509	0.4488	3.60	
5	0.563	0.4609	3.70	
6	0.619	0.4663	3.74	
7	0.665	0.4788	3.84	
8	0.719	0.4876	3.92	
9	0.779	0.4921	3.95	
10	0.826	0.5029	4.03	
11	0.874	0.5108	4.10	
12	0.926	0.5180	4.16	
13	0.974	0.5237	4.21	
14	1.047	0.5136	4.12	Nappe wetted underneath and attached to flat crest.
15	1.099	0.5174	4.15	
16	1.165	0.5177	4.15	
17	1.212	0.5225	4.19	
18	1.271	0.5238	4.21	
19	1.328	0.5260	4.22	

BAZIN'S
SERIES NO. 133

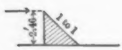


No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$	
1	0.349	0.3846	3.09	Nappe depressed.
2	0.408	0.4028	3.23	
3	0.400	0.4127	3.31	Nappe adhering.
4	0.456	0.4266	3.42	
5	0.506	0.4445	3.57	
6	0.565	0.4540	3.64	
7	0.610	0.4692	3.76	
8	0.666	0.4775	3.83	
9	0.707	0.4922	3.95	
10	0.767	0.4979	4.00	
11	0.816	0.5079	4.07	
12	0.870	0.5122	4.11	Nappe wetted underneath and attached to flat crest.
13	0.928	0.5151	4.13	
14	0.978	0.5208	4.17	
15	1.033	0.5224	4.19	
16	1.095	0.5243	4.21	
17	1.205	0.4889	3.92	Nappe wetted underneath but detached from flat crest.
18	1.283	0.4794	3.84	
19	1.276	0.4842	3.88	
20	1.346	0.4793	3.84	
21	1.415	0.4783	3.83	

BAZIN'S
SERIES No. 134

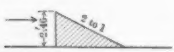
No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$	
1	0.350	0.3880	3.11	Nappe adhering.
2	0.405	0.4090	3.28	
3	0.458	0.4228	3.39	
4	0.513	0.4378	3.51	
5	0.567	0.4515	3.62	
6	0.615	0.4633	3.71	
7	0.664	0.4751	3.81	
8	0.714	0.4867	3.91	
9	0.767	0.4970	3.99	
10	0.815	0.5061	4.06	
11	0.866	0.5186	4.16	
12	0.909	0.5280	4.24	
13	0.964	0.5334	4.28	
14	1.011	0.5380	4.32	
15	1.060	0.5427	4.36	
16	1.118	0.5463	4.38	
17	1.253	0.4957	3.98	Nappe wetted underneath but detached from flat crest.
18	1.348	0.4804	3.85	
19	1.574	0.4764	3.82	
20	1.489	0.4712	3.78	

BAZIN'S
SERIES No. 136



No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.183	0.4865	3.90
2	0.244	0.4815	3.86
3	0.304	0.4803	3.85
4	0.364	0.4814	3.86
5	0.424	0.4831	3.88
6	0.484	0.4820	3.87
7	0.542	0.4842	3.88
8	0.597	0.4854	3.89
9	0.658	0.4874	3.91
10	0.713	0.4886	3.92
11	0.776	0.4900	3.93
12	0.830	0.4948	3.97
13	0.887	0.4941	3.96
14	0.953	0.4962	3.98
15	1.010	0.4950	3.97
16	1.068	0.4979	4.00
17	1.122	0.4975	3.99
18	1.179	0.4997	4.01
19	1.244	0.4993	4.01
20	1.299	0.5005	4.01
21	1.361	0.5023	4.03

BAZIN'S
SERIES No. 137



No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.268	0.4324	3.47
2	0.339	0.4310	3.45
3	0.391	0.4359	3.50
4	0.451	0.4323	3.47
5	0.513	0.4400	3.53
6	0.578	0.4377	3.51
7	0.637	0.4378	3.51
8	0.700	0.4434	3.55
9	0.765	0.4437	3.56
10	0.822	0.4441	3.56
11	0.887	0.4445	3.56
12	0.946	0.4513	3.62
13	1.012	0.4476	3.59
14	1.078	0.4505	3.61
15	1.142	0.4490	3.60
16	1.201	0.4517	3.62
17	1.262	0.4520	3.62
18	1.322	0.4543	3.64

BAZIN'S

SERIES No. 138.

No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.194	0.4450	3.57
2	0.263	0.4357	3.50
3	0.327	0.4344	3.48
4	0.391	0.4356	3.50
5	0.447	0.4429	3.56
6	0.510	0.4524	3.63
7	0.571	0.4511	3.62
8	0.626	0.4622	3.71
9	0.685	0.4565	3.66
10	0.745	0.4597	3.69
11	0.807	0.4611	3.70
12	0.873	0.4537	3.72
13	0.927	0.4643	3.72
14	0.992	0.4683	3.76
15	1.045	0.4743	3.80
16	1.110	0.4707	3.78
17	1.176	0.4716	3.79
18	1.233	0.4746	3.81
19	1.289	0.4758	3.82
20	1.355	0.4758	3.82
21	1.427	0.4778	3.83

BAZIN'S

SERIES No. 139.

No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.190	0.4561	3.66
2	0.253	0.4585	3.68
3	0.312	0.4641	3.72
4	0.375	0.4570	3.66
5	0.434	0.4651	3.73
6	0.500	0.4613	3.72
7	0.552	0.4706	3.78
8	0.615	0.4693	3.76
9	0.667	0.4764	3.82
10	0.733	0.4725	3.79
11	0.798	0.4734	3.80
12	0.852	0.4789	3.84
13	0.915	0.4808	3.86
14	0.969	0.4821	3.87
15	1.023	0.4883	3.92
16	1.092	0.4856	3.90
17	1.151	0.4868	3.90
18	1.210	0.4909	3.94
19	1.258	0.4927	3.95
20	1.326	0.4904	3.93
21	1.394	0.4898	3.93

BAZIN'S

SERIES No. 140.

No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.191	0.4696	3.77
2	0.252	0.4666	3.75
3	0.308	0.4677	3.75
4	0.371	0.4620	3.71
5	0.436	0.4699	3.77
6	0.487	0.4666	3.75
7	0.549	0.4747	3.81
8	0.604	0.4761	3.82
9	0.664	0.4783	3.83
10	0.719	0.4792	3.84
11	0.785	0.4842	3.88
12	0.837	0.4843	3.88
13	0.905	0.4889	3.92
14	0.961	0.4861	3.90
15	1.023	0.4920	3.95
16	1.080	0.4905	3.93
17	1.143	0.4951	3.97
18	1.195	0.4930	3.96
19	1.254	0.4951	3.97
20	1.316	0.4978	3.99
21	1.375	0.5000	4.01

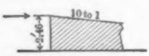
BAZIN'S

SERIES No. 141.

No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.214	0.3771	3.02
2	0.281	0.3854	3.09
3	0.355	0.3831	3.07
4	0.425	0.3790	3.04
5	0.489	0.3836	3.08
6	0.561	0.3846	3.08
7	0.624	0.3957	3.17
8	0.692	0.3882	3.11
9	0.758	0.3901	3.12
10	0.822	0.3933	3.15
11	0.888	0.3960	3.17
12	0.956	0.3977	3.19
13	1.029	0.3978	3.19
14	1.113	0.3955	3.17
15	1.165	0.3996	3.21
16	1.237	0.3987	3.20
17	1.298	0.4016	3.22
18	1.369	0.4016	3.22
19	1.431	0.4047	3.24
20	1.463	0.4051	3.25

BAZIN'S

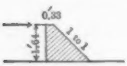
SERIES No. 142.



1	0.300	0.3537	2.83
2	0.369	0.3624	2.90
3	0.447	0.3582	2.87
4	0.509	0.3565	2.86
5	0.591	0.3594	2.88
6	0.666	0.3572	2.86
7	0.727	0.3639	2.92
8	0.795	0.3660	2.94
9	0.861	0.3661	2.94
10	0.934	0.3681	2.95
11	1.007	0.3677	2.95
12	1.079	0.3710	2.98
13	1.149	0.3716	2.98
14	1.222	0.3726	2.99
15	1.285	0.3738	3.00
16	1.362	0.3742	3.00
17	1.430	0.3752	3.01

BAZIN'S

SERIES No. 143.



1	0.352	0.3886	3.11	Nappe adhering and attached to flat crest.
2	0.415	0.3985	3.19	
3	0.462	0.4250	3.41	
4	0.513	0.4386	3.52	
5	0.567	0.4515	3.62	
6	0.619	0.4608	3.70	
7	0.675	0.4704	3.77	
8	0.725	0.4803	3.85	
9	0.771	0.4938	3.96	
10	0.821	0.5059	4.06	
11	0.874	0.5135	4.12	
12	0.920	0.5208	4.18	
13	0.972	0.5283	4.24	
14	1.022	0.5321	4.27	
15	1.072	0.5404	4.34	
16	1.126	0.5435	4.36	
17	1.172	0.5489	4.40	
18	1.227	0.5514	4.42	
19	1.331	0.5250	4.21	Nappe adhering but detached from flat crest.
20	1.369	0.5229	4.20	

BAZIN'S

SERIES NO. 144

No. of experiment	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.358	0.3808	3.05
2	0.419	0.3954	3.17
3	0.475	0.4081	3.27
4	0.533	0.4205	3.37
5	0.592	0.4255	3.41
6	0.648	0.4372	3.50
7	0.700	0.4433	3.55
8	0.757	0.4531	3.63
9	0.810	0.4588	3.68
10	0.865	0.4679	3.75
11	0.919	0.4714	3.78
12	0.978	0.4777	3.83
13	1.034	0.4774	3.83
14	1.104	0.4759	3.82
15	1.164	0.4766	3.82
16	1.232	0.4769	3.83
17	1.286	0.4784	3.84
18	1.351	0.4782	3.84
19	1.411	0.4811	3.86

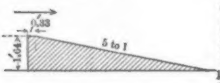
BAZIN'S

SERIES NO. 145

1	0.359	0.3765	3.02
2	0.424	0.3870	3.10
3	0.479	0.3971	3.18
4	0.547	0.4059	3.25
5	0.598	0.4177	3.35
6	0.658	0.4218	3.38
7	0.720	0.4262	3.42
8	0.781	0.4322	3.47
9	0.835	0.4358	3.49
10	0.902	0.4409	3.53
11	0.962	0.4409	3.53
12	1.032	0.4425	3.53
13	1.087	0 4461	3.58
14	1.152	0 4474	3.58
15	1.210	0.4505	3.61
16	1.274	0.4502	3.61
17	1.334	0.4515	3.62
18	1.396	0.4538	3.64
19	1.467	0.4544	3.64

BAZIN'S

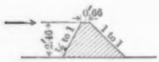
SERIES NO. 146



1	0.367	0.3632	2.91
2	0.437	0.3698	2.97
3	0.492	0.3837	3.08
4	0.567	0.3804	3.05
5	0.621	0.3923	3.14
6	0.695	0.3885	3.11
7	0.749	0.3987	3.20
8	0.817	0.3988	3.20
9	0.883	0.4021	3.22
10	0.951	0.4024	3.22
11	1.012	0.4080	3.27
12	1.085	0.4072	3.26
13	1.141	0.4119	3.30
14	1.213	0.4124	3.30
15	1.272	0.4153	3.33
16	1.344	0.4169	3.34
17	1.405	0.4190	3.36
18	1.473	0.4211	3.38
19	1.532	0.4233	3.39

BAZIN'S

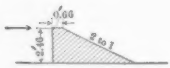
SERIES NO. 147



No. of experiment	h = observed depth on crest, in feet.	m = coefficient of discharge	$m\sqrt{2g}$
1	0.231	0.3429	2.75
2	0.308	0.3552	2.85
3	0.373	0.3566	2.86
4	0.438	0.3698	2.97
5	0.503	0.3759	3.02
6	0.569	0.3907	3.13
7	0.637	0.3999	3.20
8	0.681	0.4045	3.24
9	0.734	0.4170	3.34
10	0.797	0.4238	3.40
11	0.845	0.4299	3.44
12	0.898	0.4409	3.53
13	0.953	0.4453	3.57
14	1.015	0.4535	3.63
15	1.063	0.4580	3.67
16	1.115	0.4653	3.73
17	1.165	0.4723	3.79
18	1.217	0.4775	3.83
19	1.265	0.4803	3.85
20	1.332	0.4854	3.89
21	1.394	0.4962	3.98

BAZIN'S

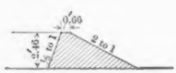
SERIES NO. 149



1	0.248	0.3158	2.55
2	0.317	0.3217	2.58
3	0.390	0.3322	2.67
4	0.455	0.3410	2.73
5	0.521	0.3513	2.82
6	0.585	0.3607	2.89
7	0.653	0.3705	2.97
8	0.705	0.3740	3.00
9	0.766	0.3841	3.08
10	0.818	0.3946	3.16
11	0.882	0.4030	3.23
12	0.942	0.4119	3.30
13	0.999	0.4193	3.36
14	1.051	0.4231	3.39
15	1.103	0.4308	3.45
16	1.165	0.4353	3.49
17	1.209	0.4396	3.52
18	1.281	0.4447	3.57
19	1.330	0.4492	3.60
20	1.385	0.4527	3.63
21	1.446	0.4580	3.67

BAZIN'S

SERIES NO. 150



1	0.248	0.3153	2.53
2	0.323	0.3297	2.65
3	0.379	0.3466	2.78
4	0.459	0.3515	2.82
5	0.512	0.3631	2.91
6	0.586	0.3692	2.96
7	0.637	0.3832	3.07
8	0.698	0.3892	3.12
9	0.751	0.3967	3.18
10	0.814	0.4063	3.26
11	0.869	0.4127	3.31
12	0.928	0.4203	3.37
13	0.982	0.4261	3.42
14	1.043	0.4328	3.47
15	1.095	0.4381	3.51
16	1.152	0.4439	3.56
17	1.215	0.4468	3.58
18	1.259	0.4529	3.63
19	1.315	0.4553	3.65
20	1.323	0.4555	3.65
21	1.380	0.4592	3.68
22	1.439	0.4645	3.73

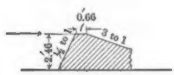
BAZIN'S

SERIES NO. 151

No. of experiment	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.201	0.3373	2.71
2	0.240	0.3508	2.81
3	0.307	0.3476	2.79
4	0.391	0.3473	2.79
5	0.445	0.3641	2.92
6	0.514	0.3679	2.95
7	0.537	0.3720	2.98
8	0.573	0.3807	3.05
9	0.643	0.3859	3.09
10	0.695	0.3990	3.20
11	0.756	0.4047	3.24
12	0.800	0.4120	3.30
13	0.826	0.4131	3.31
14	0.867	0.4191	3.36
15	0.921	0.4226	3.39
16	0.975	0.4311	3.46
17	1.027	0.4373	3.56
18	1.090	0.4395	3.52
19	1.112	0.4459	3.57
20	1.140	0.4493	3.60
21	1.209	0.4499	3.61
22	1.248	0.4546	3.64
23	1.314	0.4587	3.68
24	1.352	0.4629	3.71
25	1.416	0.4669	3.75

BAZIN'S

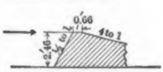
SERIES NO. 153



1	0.237	0.3408	2.73
2	0.301	0.3458	2.77
3	0.372	0.3476	2.79
4	0.373	0.3534	2.83
5	0.440	0.3613	2.90
6	0.505	0.3649	2.93
7	0.576	0.3747	3.00
8	0.637	0.3827	3.07
9	0.696	0.3866	3.10
10	0.701	0.3863	3.10
11	0.760	0.3930	3.15
12	0.762	0.3940	3.16
13	0.814	0.3996	3.20
14	0.879	0.4057	3.25
15	0.937	0.4107	3.29
16	0.993	0.4171	3.34
17	1.001	0.4159	3.33
18	1.055	0.4237	3.40
19	1.102	0.4253	3.41
20	1.170	0.4318	3.46
21	1.226	0.4341	3.48
22	1.290	0.4379	3.51
23	1.289	0.4392	3.52
24	1.347	0.4408	3.53
25	1.404	0.4463	3.58
26	1.436	0.4466	3.58

BAZIN'S

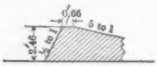
SERIES NO. 154



1	0.236	0.3370	2.70
2	0.308	0.3421	2.74
3	0.373	0.3521	2.83
4	0.447	0.3554	2.85
5	0.508	0.3677	2.95
6	0.577	0.3701	2.97
7	0.643	0.3787	3.04
8	0.706	0.3826	3.07
9	0.760	0.3950	3.17
10	0.823	0.3987	3.20
11	0.888	0.3995	3.20
12	0.946	0.4043	3.24
13	1.011	0.4095	3.28
14	1.075	0.4123	3.31
15	1.138	0.4186	3.36
16	1.195	0.4204	3.37
17	1.250	0.4245	3.40
18	1.310	0.4282	3.43
19	1.370	0.4298	3.45
20	1.430	0.4334	3.48

BAZIN'S

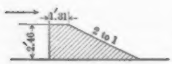
SERIES NO. 156



No. of experiment	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.245	0.3441	2.76
2	0.311	0.3488	2.79
3	0.382	0.3544	2.83
4	0.446	0.3611	2.90
5	0.508	0.3624	2.91
6	0.576	0.3675	2.95
7	0.638	0.3758	3.01
8	0.703	0.3821	3.06
9	0.764	0.3860	3.09
10	0.834	0.3897	3.12
11	0.888	0.3958	3.17
12	0.956	0.4035	3.24
13	1.018	0.4021	3.22
14	1.073	0.4119	3.30
15	1.138	0.4108	3.28
16	1.203	0.4126	3.31
17	1.269	0.4161	3.33
18	1.340	0.4194	3.36
19	1.394	0.4195	3.36
20	1.457	0.4227	3.38

BAZIN'S

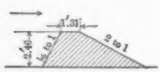
SERIES NO. 157



1	0.241	0.3282	2.63
2	0.321	0.3238	2.60
3	0.395	0 3280	2.63
4	0.475	0.3299	2.65
5	0.547	0.3288	2.64
6	0.624	0.3317	2.66
7	0.696	0.3371	2.70
8	0.765	0.3403	2.73
9	0.826	0.3470	2.78
10	0.893	0.3498	2.81
11	0.957	0.3574	2.87
12	1.030	0.3591	2.88
13	1.093	0.3655	2.93
14	1.156	0.3698	2.97
15	1.216	0.3765	3.02
16	1.277	0.3801	3.05
17	1.337	0.3855	3.09
18	1.394	0.3883	3.11
19	1.456	0.3959	3.17
20	1.474	0.3989	3.20

BAZIN'S

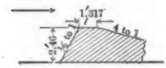
SERIES NO. 158



1	0.234	0.3479	2.79
2	0.312	0.3389	2.72
3	0.383	0.3463	2.77
4	0.457	0.3479	2.79
5	0.530	0.3499	2.81
6	0.600	0.3524	2.82
7	0.672	0.3559	2.86
8	0.733	0.3606	2.90
9	0.799	0.3633	2.91
10	0.860	0.3682	2.95
11	0.930	0.3744	3.00
12	0.984	0.3799	3.04
13	1.055	0.3866	3.10
14	1.125	0.3889	3.12
15	1.179	0.3923	3.14
16	1.243	0.3983	3.19
17	1.297	0.4025	3.22
18	1.361	0.4066	3.25
19	1.412	0.4112	3.30
20	1.457	0.4142	3.32

BAZIN'S

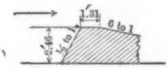
SERIES NO. 159



No. of experiment	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$.
1	0.234	0.3333	2.68
2	0.304	0.3424	2.75
3	0.379	0.3515	2.82
4	0.387	0.3507	2.82
5	0.457	0.3498	2.81
6	0.516	0.3620	2.91
7	0.526	0.3544	2.84
8	0.599	0.3519	2.82
9	0.664	0.3571	2.87
10	0.670	0.3527	2.83
11	0.735	0.3591	2.88
12	0.797	0.3662	2.94
13	0.861	0.3727	2.99
14	0.876	0.3665	2.94
15	0.935	0.3654	2.93
16	0.994	0.3751	3.01
17	1.068	0.3777	3.03
18	1.126	0.3869	3.10
19	1.145	0.3798	3.05
20	1.198	0.3839	3.08
21	1.261	0.3878	3.11
22	1.320	0.3935	3.15
23	1.332	0.3906	3.13
24	1.389	0.3912	3.14
25	1.445	0.3978	3.19
26	1.456	0.3977	3.19

BAZIN'S

SERIES NO. 160



1	0.451	0.3505	2.81
2	0.522	0.3512	2.82
3	0.593	0.3541	2.84
4	0.663	0.3589	2.88
5	0.735	0.3598	2.89
6	0.798	0.3629	2.91
7	0.863	0.3641	2.92
8	0.930	0.3701	2.97
9	0.998	0.3731	2.94
10	1.074	0.3772	3.02
11	1.129	0.3784	3.03
12	1.193	0.3818	3.06
13	1.254	0.3842	3.08
14	1.326	0.3868	3.10
15	1.389	0.3919	3.14
16	1.457	0.3937	3.16

BAZIN'S

SERIES NO. 161

1	0.298	0.5373	4.31
2	0.354	0.5357	4.30
3	0.413	0.5308	4.26
4	0.472	0.5272	4.23
5	0.529	0.5259	4.22
6	0.581	0.5297	4.25
7	0.639	0.5289	4.24
8	0.693	0.5314	4.26
9	0.750	0.5331	4.28
10	0.803	0.5374	4.31
11	0.864	0.5373	4.31
12	0.919	0.5385	4.32
13	0.960	0.5405	4.34
14	0.992	0.5359	4.30
15	1.019	0.5374	4.31
16	1.056	0.5340	4.28
17	1.083	0.5323	4.27
18	1.118	0.5282	4.24
19	1.157	0.5196	4.17
20	1.187	0.5188	4.16
21	1.225	0.5139	4.12
22	1.263	0.5102	4.09
23	1.289	0.5118	4.11
24	1.326	0.5087	4.08
25	1.359	0.5086	4.08

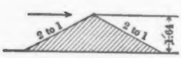
BAZIN'S

SERIES NO. 163

No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.184	0.4746	3.81
2	0.244	0.4781	3.83
3	0.303	0.4786	3.84
4	0.366	0.4775	3.83
5	0.423	0.4779	3.83
6	0.486	0.4759	3.82
7	0.536	0.4816	3.86
8	0.593	0.4917	3.94
9	0.653	0.4880	3.91
10	0.702	0.4993	4.01
11	0.769	0.4966	3.98
12	0.827	0.5017	4.02
13	0.882	0.5006	4.02
14	0.949	0.5038	4.04
15	0.998	0.5062	4.06
16	1.056	0.5060	4.06
17	1.114	0.5052	4.05
18	1.171	0.5071	4.07
19	1.231	0.5076	4.07
20	1.285	0.5129	4.12
21	1.339	0.5199	4.17

BAZIN'S

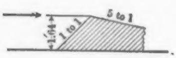
SERIES NO. 164



No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.244	0.4811	3.86
2	0.305	0.4861	3.90
3	0.367	0.4821	3.87
4	0.425	0.4867	3.90
5	0.482	0.4828	3.87
6	0.539	0.4826	3.87
7	0.592	0.4913	3.94
8	0.651	0.4915	3.94
9	0.702	0.4948	3.97
10	0.766	0.4990	4.00
11	0.817	0.5021	4.03
12	0.877	0.5046	4.05
13	0.939	0.5070	4.07
14	0.993	0.5111	4.10
15	1.052	0.5093	4.09
16	1.115	0.5136	4.12
17	1.162	0.5151	4.13
18	1.219	0.5172	4.15
19	1.277	0.5205	4.18
20	1.330	0.5217	4.19

BAZIN'S

SERIES NO. 165

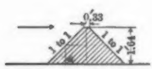


No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.337	0.4435	3.56
2	0.401	0.4447	3.56
3	0.464	0.4443	3.56
4	0.528	0.4433	3.55
5	0.593	0.4413	3.54
6	0.656	0.4422	3.54
7	0.720	0.4422	3.54
8	0.783	0.4433	3.55
9	0.843	0.4461	3.58
10	0.904	0.4506	3.61
11	0.969	0.4528	3.63
12	1.029	0.4531	3.63
13	1.090	0.4538	3.64
14	1.153	0.4553	3.65
15	1.217	0.4563	3.66
16	1.279	0.4584	3.68
17	1.341	0.4583	3.68
18	1.401	0.4601	3.69
19	1.448	0.4648	3.73

BAZIN'S
SERIES NO. 166

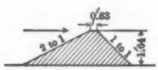
No. of experiment	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.330	0.4563	3.66
2	0.393	0.4548	3.65
3	0.455	0.4536	3.64
4	0.518	0.4538	3.64
5	0.575	0.4579	3.67
6	0.638	0.4579	3.67
7	0.699	0.4583	3.68
8	0.761	0.4596	3.69
9	0.821	0.4625	3.71
10	0.880	0.4663	3.74
11	0.947	0.4677	3.75
12	1.008	0.4681	3.76
13	1.066	0.4699	3.77
14	1.127	0.4714	3.78
15	1.187	0.4723	3.79
16	1.247	0.4731	3.80
17	1.308	0.4746	3.81
18	1.372	0.4748	3.81

BAZIN'S
SERIES NO. 167



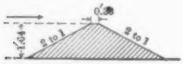
No. of experiment	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.341	0.4087	3.28
2	0.398	0.4256	3.41
3	0.453	0.4367	3.50
4	0.509	0.4541	3.64
5	0.565	0.4577	3.67
6	0.616	0.4679	3.75
7	0.671	0.4753	3.81
8	0.725	0.4823	3.87
9	0.781	0.4917	3.94
10	0.828	0.4975	3.99
11	0.885	0.5046	4.05
12	0.930	0.5127	4.11
13	0.987	0.5176	4.15
14	1.044	0.5213	4.19
15	1.084	0.5293	4.25
16	1.142	0.5319	4.27
17	1.193	0.5349	4.29
18	1.245	0.5396	4.33
19	1.304	0.5431	4.36
20	1.354	0.5437	4.36

BAZIN'S
SERIES NO. 168



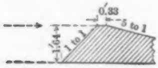
No. of experiment	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.331	0.4137	3.32
2	0.392	0.4286	3.44
3	0.444	0.4435	3.56
4	0.503	0.4505	3.61
5	0.560	0.4592	3.68
6	0.619	0.4648	3.73
7	0.669	0.4733	3.80
8	0.732	0.4740	3.80
9	0.788	0.4801	3.85
10	0.839	0.4877	3.91
11	0.898	0.4915	3.94
12	0.954	0.4949	3.97
13	1.009	0.4999	4.01
14	1.059	0.5058	4.06
15	1.117	0.5080	4.08
16	1.168	0.5122	4.11
17	1.220	0.5189	4.16
18	1.275	0.5180	4.16
19	1.338	0.5205	4.18
20	1.386	0.5252	4.22

BAZIN'S
SERIES NO. 170



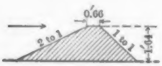
No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.351	0.4184	3.35
2	0.411	0.4300	3.45
3	0.468	0.4354	3.50
4	0.529	0.4488	3.60
5	0.585	0.4488	3.60
6	0.642	0.4576	3.67
7	0.701	0.4632	3.72
8	0.753	0.4689	3.76
9	0.811	0.4753	3.81
10	0.871	0.4796	3.85
11	0.926	0.4824	3.87
12	0.981	0.4895	3.93
13	1.009	0.4928	3.95
14	1.060	0.4948	3.97
15	1.115	0.4987	4.00
16	1.174	0.5027	4.03

BAZIN'S
SERIES NO. 172



No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.356	0.3872	3.11
2	0.419	0.4025	3.23
3	0.482	0.4039	3.24
4	0.539	0.4143	3.32
5	0.602	0.4175	3.35
6	0.661	0.4223	3.39
7	0.722	0.4255	3.41
8	0.779	0.4333	3.48
9	0.838	0.4368	3.50
10	0.906	0.4414	3.54
11	0.962	0.4436	3.56
12	1.026	0.4460	3.58
13	1.086	0.4482	3.59
14	1.152	0.4499	3.61
15	1.204	0.4558	3.66
16	1.267	0.4586	3.69
17	1.325	0.4607	3.70
18	1.377	0.4670	3.75
19	1.449	0.4645	3.73

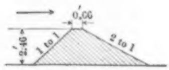
BAZIN'S
SERIES NO. 175



No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.360	0.3950	3.17
2	0.421	0.4061	3.25
3	0.487	0.4054	3.25
4	0.548	0.4119	3.30
5	0.608	0.4220	3.38
6	0.667	0.4281	3.43
7	0.726	0.4337	3.48
8	0.782	0.4396	3.52
9	0.837	0.4482	3.59
10	0.894	0.4554	3.65
11	0.949	0.4607	3.70
12	1.007	0.4649	3.73
13	1.061	0.4710	3.78
14	1.123	0.4747	3.81
15	1.174	0.4790	3.84
16	1.230	0.4811	3.86
17	1.287	0.4845	3.89
18	1.347	0.4869	3.91
19	1.387	0.4908	3.94

BAZIN'S

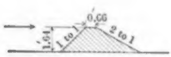
SERIES NO. 176



No. of experiment	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.237	0.3439	2.76
2	0.296	0.3413	2.74
3	0.365	0.3636	2.92
4	0.439	0.3684	2.95
5	0.494	0.3795	3.04
6	0.565	0.3867	3.10
7	0.618	0.3977	3.19
8	0.682	0.4021	3.22
9	0.733	0.4104	3.29
10	0.797	0.4175	3.35
11	0.861	0.4251	3.41
12	0.910	0.4296	3.45
13	0.974	0.4376	3.51
14	1.027	0.4409	3.53
15	1.088	0.4449	3.57
16	1.139	0.4518	3.62
17	1.196	0.4547	3.65
18	1.248	0.4586	3.68
19	1.303	0.4652	3.73
20	1.355	0.4676	3.75
21	1.420	0.4727	3.80

BAZIN'S

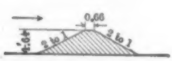
SERIES NO. 177



No. of experiment	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.367	0.3863	3.10
2	0.433	0.3872	5.18
3	0.497	0.3993	3.20
4	0.556	0.4050	3.25
5	0.617	0.4127	3.31
6	0.680	0.4180	3.35
7	0.739	0.4242	3.40
8	0.794	0.4309	3.46
9	0.850	0.4389	3.52
10	0.906	0.4482	3.59
11	0.965	0.4562	3.66
12	1.019	0.4621	3.71
13	1.077	0.4658	3.74
14	1.129	0.4716	3.79
15	1.184	0.4753	3.81
16	1.243	0.4779	3.83
17	1.296	0.4840	3.88
18	1.346	0.4867	3.91
19	1.386	0.4924	3.95

BAZIN'S

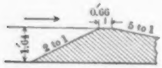
SERIES NO. 179



No. of experiment	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.364	0.3930	3.15
2	0.431	0.3997	3.20
3	0.490	0.4054	3.25
4	0.558	0.4091	3.28
5	0.614	0.4180	3.35
6	0.674	0.4244	3.40
7	0.732	0.4277	3.43
8	0.790	0.4331	3.47
9	0.851	0.4399	3.53
10	0.908	0.4480	3.59
11	0.968	0.4536	3.64
12	1.023	0.4588	3.68
13	1.078	0.4641	3.72
14	1.134	0.4656	3.74
15	1.190	0.4703	3.77
16	1.247	0.4740	3.80
17	1.298	0.4796	3.85
18	1.357	0.4840	3.88
19	1.403	0.4884	3.92

BAZIN'S

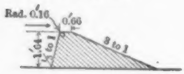
SERIES NO. 181



No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.219	0.3789	3.04
2	0.290	0.3858	3.09
3	0.356	0.3863	3.10
4	0.423	0.3900	3.13
5	0.486	0.3993	3.20
6	0.554	0.3970	3.18
7	0.621	0.4030	3.23
8	0.677	0.4092	3.28
9	0.743	0.4105	3.29
10	0.797	0.4153	3.33
11	0.861	0.4210	3.38
12	0.920	0.4267	3.42
13	0.983	0.4311	3.46
14	1.044	0.4345	3.49
15	1.111	0.4394	3.52
16	1.159	0.4413	3.54
17	1.228	0.4447	3.57
18	1.278	0.4484	3.59
19	1.342	0.4518	3.62
20	1.403	0.4534	3.63
21	1.463	0.4578	3.67

BAZIN'S

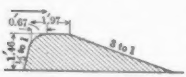
SERIES NO. 182



1	0.280	0.3927	3.15
2	0.403	0.4012	3.22
3	0.526	0.4150	3.33
4	0.641	0.4309	3.46
5	0.761	0.4448	3.56
6	0.867	0.4584	3.68
7	0.989	0.4654	3.73
8	1.091	0.4749	3.81
9	1.210	0.4821	3.87
10	1.320	0.4870	3.91
11	1.413	0.4966	3.99

BAZIN'S

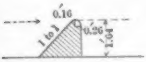
SERIES NO. 188



1	0.289	0.3758	3.01
2	0.425	0.3835	3.08
3	0.563	0.3836	3.08
4	0.697	0.3925	3.15
5	0.825	0.3982	3.19
6	0.953	0.4045	3.24
7	1.080	0.4120	3.30
8	1.201	0.4193	3.36
9	1.322	0.4274	3.43
10	1.431	0.4388	3.52
11	1.546	0.4432	3.55

BAZIN'S

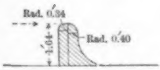
SERIES NO. 189



No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge	$m\sqrt{2g}$	
1	0.248	0.4605	3.69	Adhering Nappe.
2	0.300	0.4334	3.88	
3	0.355	0.4967	3.99	
4	0.406	0.5090	4.08	
5	0.457	0.5198	4.17	
6	0.511	0.5331	4.28	
7	0.557	0.5389	4.32	
8	0.607	0.5517	4.43	
9	0 655	0.5563	4.46	
10	0.706	0.5635	4.52	
11	0.706	0.5624	4.51	Nappe wetted underneath.
12	0.756	0.5664	4.54	
13	0.811	0.5706	4.58	
14	0.864	0.5698	4.57	
15	0.923	0.5677	4.55	
16	0.979	0.5681	4.56	
17	1.042	0.5644	4.53	
18	1.097	0.5612	4.50	
19	1.151	0.5629	4.52	
20	1.227	0.5550	4.45	
21	1.274	0.5572	4.47	

BAZIN'S

SERIES NO. 193



1	0.260	0.4354	3.49
2	0.317	0.4496	3.61
3	0.374	0.4635	3.72
4	0.429	0.4716	3.79
5	0.478	0.4891	3.92
6	0.534	0.4924	3.95
7	0.585	0.5049	4.05
8	0.634	0.5108	4.10
9	0.684	0.5238	4.20
10	0.736	0.5293	4.25
11	0.780	0.5418	4.35
12	0.836	0.5480	4.40
13	0.886	0.5559	4.46
14	0.932	0.5628	4.52
15	0.979	0.5644	4.53
16	1.035	0.5710	4.58
17	1.093	0.5675	4.55
18	1.148	0.5663	4.54
19	1.201	0.5695	4.57
20	1.266	0.5665	4.54

General Résumé of Bazin's Experiments.

This chapter has also been translated from Bazin's final paper in *Annales des Ponts et Chaussées* for 1898.*

Classification of the Different Species of Nappes.—When water is discharged through a submerged orifice, it is well known that the discharge Q varies according to the nature of the orifice. The coefficient m, in the formula $Q = m\, c \sqrt{2\, g\, h}$, must be determined for each particular case. This coefficient varies, however, only within relatively narrow limits.

The conditions in the case of a weir are much more complex. The influence of the shape (width of crest, degree of inclination of the up-stream and down-stream faces of the weir, etc.) always enters, but other factors conspire to make the value of the coefficient m, in the formula $Q = m\, l\, h \sqrt{2\, g\, h}$, vary within much wider limits than for discharge through an orifice.

The sloping nappe of a weir differs from the vein issuing from an orifice in that it may assume a variety of perfectly distinct forms. These forms constitute, in reality, as many distinct cases, each of which it is necessary to study separately, since, by confusing them one must necessarily expose himself to serious errors. It is proper, at the start, to classify these different forms and to make known their distinctive characteristics.

Free Nappes.—The most simple and definite case is that of a sharp-crested weir without lateral contractions, in which the nappe falls

* Bazin's formulas have been changed, so as to make English measures applicable, by the introduction of a conversion factor when necessary. The mathematical symbols are as follows:

h = head on crest of weir, in feet;
u = mean velocity in channel above the weir, in feet;
a = a constant (replaces α used by Bazin);
K = a constant;
g = acceleration of gravity, = 32.2;
l = length of crest of weir, in feet;
n = a constant (replaces μ used by Bazin);
p = height of crest of weir above bottom of channel, in feet;
m = coefficient of discharge over a given weir;
m' = corresponding coefficient for a standard weir;
Q = discharge, in cubic feet per second;
h_l = difference of elevation of crest of weir and of water on the down-stream side, in feet, when the latter is below the crest of the weir;
h_e = difference of elevation of crest of weir and of water on the down-stream side, in feet, when the weir is drowned;
c = width of crest of a flat-crested weir, in feet;
R = radius of curvature of a weir rounded at the back, in feet;
z = $h - h_e$, in feet.

freely in the air, its lower surface always subject to the pressure of the atmosphere. The lateral contraction may be suppressed by making the length of the weir equal to the width of the canal leading to it, which is supposed to have vertical walls. Immediately below the crest, these walls should be constructed in such a manner as to permit free access of air beneath the nappe, which then represents a portion of a nappe of indefinite length. The resulting phenomena of flow are perfectly constant; the nappe, independent of any influence of the water below the weir, permits of a precise determination of the coefficient m.

Depressed Nappes and Nappes Wetted Underneath.—When the walls of the canal in which the weir is placed are not constructed in such a manner as to maintain free access of air beneath the nappe, the phenomena of discharge become more complicated, and the form of the nappe, remarkably constant in the case where it falls freely over the weir, undergoes considerable modification, according to the amount of discharge.

When the head does not exceed a certain limit, the nappe remains detached from the face of the weir, imprisoning underneath it a volume of air at a pressure inferior to that of the external atmosphere. At the same time the water in the space between the foot of the nappe and that of the weir rises to a level above that of the stream below the weir. We have, then, a species of free nappe, modified and depressed by the excess of external pressure. The discharge over such a sharp-crested weir is, for equal heads, slightly greater than over a true free nappe. The difference increases as the volume of the imprisoned air diminishes.

Depressed nappes are not very stable; the accidental entrance of air from time to time produces variations in both the interior pressure and the discharge. When the air has entirely disappeared, the nappe takes a more definite form, which may be designated as wetted underneath. The overflowing nappe encloses a small portion of turbulent water, which does not partake of the translatory movement of the vein, properly speaking.

The occurrence of a nappe wetted underneath may be independent of the level of the water on the down-stream side of the weir; or, on the contrary, it may be influenced by this level, every modification of which then reacts on the discharge over the weir.

The former is the case when the overfall is followed by a rapid which terminates in an abrupt ressault, below which the flow takes place in accordance with the condition of the channel. The position of the ressault is without influence on the discharge, provided it does not enclose the foot of the nappe.

In the second case, that is to say, when the foot of the nappe is more or less enclosed in the water in the channel below the weir, it is not easy to separate the influence of the water in the channel, for the discharge may be modified, although this water does not rise to the level of the crest of the weir.

Undulating Nappes.—When the level of the water in the channel below the weir is raised to the height of the weir crest, the wetted nappe retains its form as long as the difference in level, or fall of the water from the up-stream to the down-stream side of the weir, does not descend below a certain limit. Its characteristic profile persists, although in part concealed by its immersion in the channel on the down-stream side, but, where the fall or difference in level is progressively diminished, a moment comes when the nappe returns suddenly to the surface, undulating in the meanwhile. This change, although very apparent, does not exert any important influence on the value of the coefficient of discharge.

Adhering Nappes.—The forms of nappes thus far considered are those most ordinarily encountered. Another form exists, which appears under certain conditions, in which the nappe, instead of enfolding a small mass of turbulent water having no translatory movement, as in the case of the nappe wetted underneath, is, on the contrary, in close contact with the face of the weir. It presents then, in certain cases, some interesting particulars, and to this remarkable form there often corresponds a considerable increase of the coefficient of discharge.

The *ensemble* of the phenomena of discharge is very complex, and one cannot often determine the discharge of a weir with precision without previously knowing under which particular form of nappe the discharge took place. Taking, for example, a sharp-crested weir 0.75 m. (2.46 ft.) high, we have shown that for the same head of 0.20 m. (0.656 ft.) the nappe may assume four very distinct forms, to which correspond the following different values of the coefficient m in the formula $Q = m\,l\,h\,\sqrt{2\,g\,h}$:

	$m.$	$m\sqrt{2g}.$
(1) Free nappe, under surface always subjected to atmospheric pressure	0.433	3.47
(2) Depressed nappe, imprisoning a certain volume of air at a pressure below that of the atmosphere	0.460	3.69
(3) Nappe wetted underneath, no air imprisoned, level of water on down-stream side 0.125 m. (0.42 ft.) below crest of weir	0.497	3.99
(4) Adhering nappe, the ressault being at a distance from the foot of the nappe, which is completely exposed	0.554	4.45

Sharp-Crested Weirs.

Free Nappes.—When the nappe, in flowing over a sharp-crested weir, has its lower surface in free communication with the atmosphere, the only element which modifies the coefficient m is the mean velocity u in the channel leading to the weir. In order to take this into consideration in the formula, h is replaced by $h + a\frac{u^2}{2g}$. The formula becomes then, representing by n the modified coefficient m,

$$Q = n\,l\left(h + a\frac{u^2}{2g}\right)\sqrt{2g\left(h + a\frac{u^2}{2g}\right)}$$

$$= n\,l\,h\sqrt{2gh}\left(1 + a\frac{u^2}{2gh}\right)^{\frac{3}{2}}$$

or, approximately, considering that $\frac{u^2}{2gh}$ is a fraction rarely exceeding a few centimeters,

$$Q = n\,l\,h\sqrt{2gh}\left(1 + \frac{3}{2}\,a\frac{u^2}{2gh}\right) \quad \text{.............. (1)}$$

This expression is not in a form convenient for use, since the velocity u depends on the discharge to be determined. If p be used to designate the height of the weir above the bottom of the channel, the wetted section of the canal is $l\,(h + p)$, and we have

$$\frac{u^2}{2g} = \frac{Q^2}{2gl^2(h+p)^2}$$

or, simply replacing Q by its value $m\,l\,h\sqrt{2gh}$, we have,

$$\frac{u^2}{2gh} = m^2\left(\frac{h}{h+p}\right)^2$$

Letting, for short $\left(\frac{3}{2} a m^2\right) = K$, the above expression takes the more practical form

$$Q = n\left[1 + K\left(\frac{h}{h+p}\right)^2\right] l h \sqrt{2gh} \dots\dots\dots \quad (2)$$

We have determined the coefficients a, K and n by comparative experiments on five weirs of different heights. a and K are not perfectly constant, but one may take, as a mean, $a = \frac{5}{3}$; $K = 0.55$. As to n, its value, which corresponds to the limiting case of no velocity of approach, cannot be measured directly, since it is impossible to completely suppress this velocity. Its influence, however, becomes insignificant in a high weir. The coefficient n decreases slowly as the head increases, as shown below:

Head, in feet	= 0.164, 0.328, 0.656, 0.984, 1.312, 1.640.
Corresponding values of n	= 0.448, 0.432, 0.421, 0.417, 0.414, 0.412.

When h is over 0.328 ft. its value is represented with sufficient precision by the formula,

$$n = 0.405 + \left(\frac{0.00984}{h}\right) \dots\dots\dots \quad (3)$$

Adopting for K the value 0.55, Formula (2) becomes

$$m = n\left[1 + 0.55\left(\frac{h}{h+p}\right)^2\right] \dots\dots\dots \quad (4)$$

In most cases, where the head falls between 0.10 m. (0.328 ft.) and 0.30 m (0.984 ft.), n may be taken at the constant value 0.425, and taking $K = \frac{1}{2}$ simply, the expression for m becomes

$$\left.\begin{aligned} m &= 0.425\left[1 + \frac{1}{2}\left(\frac{h}{h+p}\right)^2\right] \\ &= 0.425 + 0.212\left(\frac{h}{h+p}\right)^2 \end{aligned}\right\} \dots\dots\dots \quad (5)$$

which will answer for all practical cases where errors of 2 to 3% are permissible. It is this coefficient of discharge m, perfectly determined by the head h and the height p of the weir, which has been used for comparison. Instead of considering on the other weirs the absolute values of the coefficient m, we have compared them with the coefficient m' for a free nappe, for the same head on a sharp-crested weir of the same height.

This substitution of the ratio $\frac{m}{m}$, for the absolute values of m eliminates, in a large measure, at least, the influence of velocity of approach, and facilitates greatly the discussion of results.

In what precedes, the up-stream face of the weir has been supposed to be a vertical plane. If it is inclined, the values of the coefficient m will be modified. The coefficient is diminished when the plane of the dam is inclined up stream, but, if, on the contrary, the plane of the dam is inclined down stream, the coefficient increases to a maximum which corresponds nearly to an inclination of 30° (equals a slope of $1\frac{3}{4}:1$ on the back). The ratio between the coefficients corresponding to two different inclinations is sensibly constant for all heads, so that one may obtain the coefficient m for a sharp-crested weir at any inclination by multiplying by a constant, or modulus, the corresponding coefficient for a vertical weir, as is indicated in Table No. 3.

This ratio increases regularly from an inclination of 45° toward the up-stream side, to approximately 30° toward the down-stream side where the maximum occurs, from which point the discharge does not take place in the normal manner, since the liquid vein in its passage over the crest, instead of being freely contracted, is guided by the incline of the weir on which it rests in immediate contact.

TABLE No. 3.

		Modulus by which to multiply the coefficient for a vertical weir.
Up-stream inclination of the weir.	1 horizontal to 1 vertical	0.93
	2 " " 3 "	0.94
	1 " " 3 "	0.96
Vertical weir		1.00
Down-stream inclination of the weir.	1 horizontal to 3 vertical	1.04
	2 " " 3 "	1.07
	1 " " 1 "	1.10
	2 " " 1 "	1.12
	4 " " 1 "	1.09

Sharp-Crested Weirs. Nappes Depressed and Wetted Underneath.—When the air is not admitted freely underneath the nappe, the phenomena become more complicated. The nappe may be either depressed, as a result of air being imprisoned underneath at less than atmospheric pressure, or it may be wetted underneath without

containing any air. The discharge for a depressed nappe is slightly in excess of that for a free nappe. The difference may rise to nearly 10% at the moment when the nappe, the confined air being nearly exhausted, is at the point of assuming the form of a nappe wetted underneath. The accidental entrance of air from time to time may vary the discharge a little. The wetted nappes are more uniform. It is important to distinguish two cases according as the ressault, which is produced below the nappe, is at a distance from its foot, or partly encloses it.

First Case. Ressault at a Distance.—The coefficient m may be deduced from the coefficient m' for a free nappe by the relation

$$m = m' \left(0.878 + 0.128 \frac{p}{h}\right) \qquad (6)$$

The ratio $\frac{p}{h}$ can only have certain values, as experience has shown that it does not exceed 2.5, because the form of nappe wetted underneath does not continue if the head is less than 0.4 p. For the maximum value $\frac{p}{h} = 2.5$, we have, very nearly, $m = 1.20\ m'$; and when $h = p$, m becomes sensibly equal to m'. Finally, m is a little greater than m' when h surpasses p.

If the above formula be applied to weirs of different heights, it may be shown that for the same value of $\frac{p}{h}$ the absolute values of m do not differ greatly from those given by the equation

$$m = 0.470 + 0.0075 \frac{p^2}{h^2} \qquad (7)$$

which permits us to find an absolute value of the coefficient m without using the ratio $\frac{m}{m'}$.

Second Case. The Ressault Enclosing Part of the Nappe.—It is necessary to take into account the level of the water below the weir, and, if we designate by h_t the difference of level of the crest of the weir and of the water below, the value of m becomes

$$m = m' \left[1.06 + 0.16 \left(\frac{h_t}{p} - 0.05\right) \frac{p}{h} \right] \qquad (8)$$

In this formula, h_t is to be taken as minus when the level of the water on the down-stream side is below the crest, and as plus when it is above the crest. The formula can only be applied within certain limits of

h_l. If the difference in the level of the water above and below the weir be increased, a moment comes when the ressault is driven back from the foot of the nappe until it ceases to enclose it and changes then to the preceding case. This pushing back of the ressault takes place when the total fall $(h + h_l)$ is approximately equal to $\frac{3}{4}p$. That is to say, for a given head h the greatest admissible value of h_l is $\left(\frac{3}{4}p - h\right)$. On the other hand, when the head h is insufficient to throw back the ressault, it is necessary that the level below the weir be sufficient to sustain the foot of the nappe and to prevent the introduction of air, which would cause the nappe to return to the depressed form. The preceding formula may be simplified by suppressing the small term, 0.55, in the parentheses, and, for compensation, slightly diminishing the two other coefficients. It then becomes

$$m = m' \left(1.05 + 0.15 \frac{h_l}{h}\right) \qquad (9)$$

Sharp-Crested Weirs. Adhering Nappes.—The nappe may also take, though very rarely, a particular form, the production of which depends on the width of the dam and the form of the upper part supporting the sharp crest. The nappe becomes completely attached to the downstream face of the dam without the interposition of air. The coefficient of discharge then becomes very large and may rise as high as 1.30 m', which corresponds to an absolute value of the coefficient $m = 0.55$ or 0.56. Adhering nappes present curious particulars, but as they only occur exceptionally in practice, we may simply refer to the special studies made of them, included in *Annales des Ponts et Chaussées* for 1891.

Beam Weirs. Free Nappes.—Beam weirs are constructed of square timbers of the same cross-sectional dimensions, placed one upon another to the desired height. The back and front faces of the weir are vertical planes, but the crest, instead of being reduced to a sharp edge, presents a horizontal surface, the width of which equals the thickness of the timbers. This circumstance completely modifies the conditions of discharge, and, while this form of weir is readily constructed, it may, unfortunately, give considerable error in the gaugings.

The free nappes appear under two distinct forms, according as the

nappe is in contact with the horizontal crest, or becomes detached at the back edge in such a manner as to flow over the crest without touching the down-stream edge. In the second case the influence of the flat crest evidently disappears and the discharge is like that over a sharp-crested weir. The nappe assumes this form when the head exceeds twice the width of c of the crest, measured in the direction of discharge, but it may occur whenever the head exceeds $\frac{3}{2}c$. Between these limits the nappe is in a state of instability; it tends to detach itself from the crest and may do so under the influence of any external disturbance, as, for example, the entrance of air or the passage of a floating body over the weir.

When the nappe adheres to the crest, the coefficient m depends chiefly on the ratio $\frac{h}{c}$ and may be represented by the formula,

$$m = m' \left(0.70 + 0.185 \frac{h}{c}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

m varies, as a consequence, very rapidly. We have,

When $\frac{h}{c} = 0.50$	$\frac{m}{m'} = 0.79$	
" $= 1.00$	" $= 0.88$	
" $= 1.50$	" $= 0.98$	or 1.0 if the nappe is
" $= 2.00$	" $= 1.07$	detached.

When $\frac{h}{c}$ exceeds 2.00, $\frac{m}{m'} = 1.00$. It will be seen that between $h = \frac{3}{2}c$ and $h = 2c$, $\frac{m}{m'}$ may vary from 0.98 to 1.07, or nearly a tenth in value, or, it may remain constantly equal to unity, according as the nappe is attached to or free from the crest.

Very Wide Crests.—When the width of the crest is considerable, 1 or 2 m., for example, the foregoing formula is still applicable, giving results within a few per cent. The value of $\frac{h}{c}$ then reduces to a few tenths and the ratio $\frac{m}{m'}$ also becomes much smaller, so that m may not exceed 0.35. For example, at a head of 0.45 m. (1.476 ft.) on a weir with a flat crest 2 m. wide, $\frac{m}{m'} = 0.755$, which corresponds to an absolute value of $m = 0.337$. The formula gives $\frac{m}{m'} = 0.732$ and as a result, $m = 0.326$.

Effect of Rounding the Crest at the Back.—A slight rounding of the back edge of the crest very sensibly modifies the discharge. Fteley and Stearns have shown that rounding the back edge of the crest to a radius R augments the discharge by an amount equal to that given by a head increased by 0.7 R. This is equivalent to increasing the coefficient m in the ratio of $h^{\frac{3}{2}}$ to $(h + 0.7\ R)^{\frac{3}{2}}$ or nearly in the ratio of 1 to $1 + \frac{R}{h}$. The radius R in their experiments did not exceed 0.039 ft., and it is clear that this approximate mode of correction will not apply to cases where the radius is notably greater. We have experimented on two weirs, respectively 0.80 m. (2.624 ft.) and 2.00 m. (6.56 ft.) in width, with crests rounded at the back to a radius of 0.10 m. (0.328 ft.), and this modification has had the effect of increasing the discharge 14% on the first of these weirs and 12% on the second. A simple rounding of 1 or 2 cm. radius, such as results from wear on timbers of ordinary dimensions, is by no means negligible from the point of view of the resulting discharge.

A weir with a crest 2 m. wide, and rounded at the back, gave, for the greatest head used in the experiments, $m = 0.373$, a value differing little from that indicated by theory for the case of a nappe flowing in filaments parallel to the horizontal surface of the crest. This hypothesis may not, however, be realized experimentally in more than a very imperfect manner, as the surface of the nappe undulates continually.

Beam Weirs. Nappes Depressed and Wetted Underneath.—The depressed nappes do not differ greatly from the free. The coefficient is at first less than for a free nappe, but approaches it progressively in value and finally exceeds it slightly. It differs in this respect from a sharp-crested weir, for which the coefficient for a depressed nappe is always superior to that for a free nappe. It makes no difference, as to this, whether the nappe clings to the flat crest or is detached from it. In either case the coefficient differs little from that for a sharp-crested weir. The effect of adherence to the flat crest appears again for a nappe wetted underneath, with this added difficulty, that the moment of detachment underneath the water is not apparent, and does not correspond to any constant value of $\frac{h}{c}$. In other words, it may take place either preceding or following the formation of the

nappe wetted underneath. It is necessary, in this regard, to distinguish two cases according as the height p of the dam is greater or less than about five times the width c of the crest. When p is greater than 5 c, the nappe detaches itself from the flat crest before it becomes wetted underneath, and in the intermediate state does not differ greatly from that for a sharp-crested weir. When, on the other hand, p is less than 5 c, the nappe does not detach itself from the flat crest before becoming wetted underneath, but is very unstable at the moment of this transformation.

So long as the nappe adheres to the crest, this influence predominates, and Formula (10) is most nearly applicable to the nappe wetted underneath. On the other hand, when the nappe is detached from the flat crest, the conditions of discharge approach more nearly those for a sharp crest, to which Formula (6) may be applied. The two formulas give the same value when the head exceeds a certain limit:

$$h_l = \frac{c}{2}\left(1 + \sqrt{\frac{3p}{c}}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

For heads less than h_l, Formula (10) gives values of m slightly too small, never differing from those of the experiments, however, by more than 3 or 4 per cent. When the head exceeds h_l, one must take recourse to the other formula, although it, likewise, gives values which are too small. The difference, rather more important in this case, attains 8% as a maximum, after which it diminishes rapidly for increased heads. This maximum corresponds to the moment when the nappe is at the point of detaching itself from the flat crest. After it has become detached, the influence of width of crest disappears and Formula (6) applies with a very close degree of approximation.

It will be seen that the flat crest has the effect of doubling each species of nappe, in that two formulas must be applied according as the nappe clings to or is detached from the flat surface.

Weirs with Wide Crests, and Slope on the Faces.—The phenomena of discharge become much more complex for weirs, such as are often found in practice, with slopes of greater or less inclination on the front and back faces. The influence of the flat crest, which exerts itself in a weir built up of square timbers, is joined to that of the slope of the faces. The inclination of the up-stream face, by reduc-

ing the contraction, has the effect of increasing the discharge. While that of the down-stream face has, ordinarily, the same effect as increasing the width of the flat crest, that is to say, it diminishes the discharge. The coefficient m, then, in each particular case, depends not only on the head, but on the width of the crest and the degree of inclination of the faces. It is, therefore, exceedingly variable, and each type demands a special study.

Rounding the back edge of the crest reduces the contraction considerably and may increase the value of m 10 or 15 per cent. Considering, finally, the class of weirs with completely curved profiles, such as are occasionally encountered in hydraulic practice, the value of m may attain a relatively high figure. The coefficients for such cases have not been arranged in comparative tables, but enough particular cases are given to serve as a guide in practice. It is clearly impossible to establish a general formula which will take account of all the elements that enter to affect the discharge.

Drowned Weirs.—We have given, in discussing the experiments on sharp-crested weirs drowned by the water on the down-stream side, two formulas; one of which applies to cases where the weir is not deeply drowned. The other, which is more general in its application, is

$$m = m'\left(1.08 + 0.18\,\frac{h_e}{p}\right)\sqrt[3]{\frac{z}{h}} \quad \text{.......………} (11)$$

The two formulas mentioned have been so established as to represent in the best possible manner the experiments from which they have been deduced. In cases where a less precise approximation will suffice, Formula (11) may be made applicable, by altering slightly its coefficients, as follows:

$$m = 1.05\,m'\left[1 + \frac{1}{5}\left(\frac{h_e}{p}\right)\right]\sqrt[3]{\frac{z}{h}} \quad \text{..............} (12)$$

This new expression is practically equivalent to the two others and gives the same values within 1 or 2%, except when the ratios $\frac{h}{p}$ and $\frac{h_1}{p}$ are very small. The difference may then be as much as 4 or 5%, but in this case the determination of the coefficient m is always very uncertain.

The effect of drowning is not the same for a wide-crested weir.

Raising the plane of the water below the weir, which in the case of a sharp-crested weir affected the flow on the up-stream side before the water below had reached the height of the crest, does not commence to take effect on a wide-crested weir until after the level of the water on the down-stream side is considerably above the crest; and the greater the width of the crest, the less is its ultimate effect. In our experiments on a crest 2 m. in width we have shown that the water on the down-stream side must rise to a height above the crest equal to $\frac{5}{6}$ of the head h before it affects the level on the up-stream side.

When the plane surface, which forms the crest of a weir, is very wide, it constitutes a sort of channel, and, in a measure, as the length of this channel is increased, the conditions of discharge depart from those which pertain to a weir, properly speaking, and approach those for a channel with a horizontal bottom.

The Cornell University Experiments.

At the beginning of the study of Bazin's work it was the writer's opinion that his coefficients could be fairly extended to depths on the crest of about 4 ft. without material error, and on this basis a number of discharge curves were worked out in the manner to be described. On further study, however, it seemed probable that some of Bazin's Series, especially Nos. 130 and 135 and a few others, might be somewhat too high for deep flows, for the reason that at depths on the crest from 0 up to about 0.6 to 1.0 ft. the nappes were depressed and adherent, and above 0.6 to 1.0 ft. were wetted underneath, thus indicating that probably the conditions of the experiments were such as not to insure the free admission of air beneath the nappes, this condition leading to higher flows than with air freely admitted. Or, on the other hand, as Bazin himself points out, the limit of perfect detachment may not have been reached in his experiments.

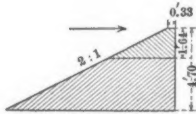
Again, Bazin's weirs were constructed with closed fronts, thus offering an opportunity for adhering nappes, while in actual practice, for sections corresponding to Series Nos. 130 and 135, the water generally flows over a lip, the nappe dropping into a free air space below. This general condition is illustrated by the dams shown on several of the illustrative figures following. The conditions at the ends of such dams are such as to usually permit the free admission of air.

CORNELL UNIVERSITY
EXPERIMENT NO. 1
BAZIN'S SERIES NO. 130
MAY 20TH, 1899.

Length of crest 6.58 ft.

Number of experiments, 7.
Limiting heads, 1.19 ft. and 4.97 ft.

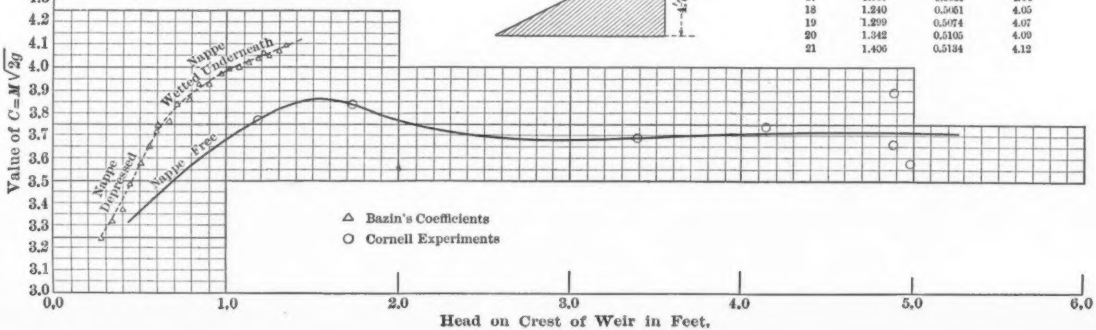
Head, in feet. H	M	$C=M\sqrt{2g}$	Q per foot.
0.5	0.4180	3.355	1.20
1.0	0.4588	3.680	3.44
1.5	0.4764	3.820	6.59
2.0	0.4698	3.770	10.66
2.5	0.4598	3.690	14.59
3.0	0.4585	3.680	19.13
3.5	0.4598	3.690	24.25
4.0	0.4616	3.705	29.68
4.5	0.4625	3.710	35.40
5.0	0.4625	3.710	41.46
5.5	0.4625	3.710	47.85
6.0	0.4625	3.710	54.54



Bazin Section

BAZIN'S
SERIES NO. 130

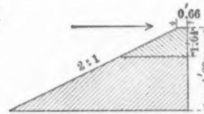
No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$	
1	0.274	0.4037	3.24	Nappe depressed or adhering, but unstable.
2	0.336	0.4138	3.32	
3	0.396	0.4213	3.37	
4	0.452	0.4339	3.48	
5	0.510	0.4458	3.57	
6	0.565	0.4546	3.65	
7	0.618	0.4663	3.74	Nappe wetted underneath and attached to flat crest.
8	0.672	0.4694	3.76	
9	0.728	0.4769	3.83	
10	0.785	0.4801	3.85	
11	0.837	0.4882	3.92	
12	0.898	0.4898	3.92	
13	0.956	0.4949	3.97	
14	1.011	0.4970	3.99	
15	1.069	0.4988	4.00	
16	1.123	0.5011	4.02	
17	1.187	0.5031	4.04	
18	1.240	0.5061	4.05	
19	1.299	0.5074	4.07	
20	1.342	0.5105	4.09	
21	1.406	0.5134	4.12	



CORNELL UNIVERSITY
EXPERIMENT NO. 2.
BAZIN'S SERIES NO. 135.
MAY 24TH, 1899.

Length of crest 6.58 ft.
Number of experiments, 7.
Limiting heads, 0.341 ft. and 5.05 ft.

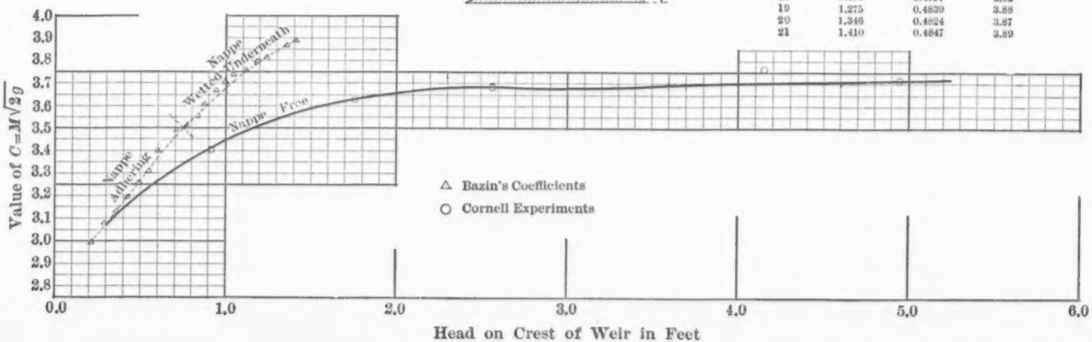
Head, in feet. H	M	$C=M\sqrt{2g}$	Q per foot.
0.5	0.4012	3.220	1.15
1.0	0.4285	3.440	3.44
1.5	0.4474	3.590	6.59
2.0	0.4564	3.663	10.35
2.5	0.4598	3.690	14.59
3.0	0.4585	3.680	19.12
3.5	0.4598	3.690	24.15
4.0	0.4615	3.705	29.65
4.5	0.4625	3.710	35.40
5.0	0.4625	3.710	41.46
5.5	0.4625	3.710	47.84
6.0	0.4625	3.710	54.54



Bazin Section

BAZIN'S
SERIES NO. 135.

No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$	
1	0.224	0.3727	2.99	Nappe depressed.
2	0.292	0.3839	3.08	
3	0.353	0.3912	3.13	
4	0.422	0.3981	3.19	
5	0.480	0.4071	3.26	
6	0.541	0.4127	3.31	
7	0.597	0.4243	3.40	
8	0.657	0.4264	3.42	Nappe wetted underneath.
9	0.712	0.4347	3.49	
10	0.773	0.4394	3.52	
11	0.829	0.4439	3.55	
12	0.892	0.4496	3.61	
13	0.939	0.4575	3.67	
14	1.011	0.4614	3.70	
15	1.063	0.4657	3.74	
16	1.119	0.4695	3.76	
17	1.175	0.4740	3.80	
18	1.230	0.4757	3.82	
19	1.275	0.4839	3.88	
20	1.346	0.4824	3.87	
21	1.410	0.4847	3.89	



CORNELL UNIVERSITY

EXPERIMENT NO. 3.

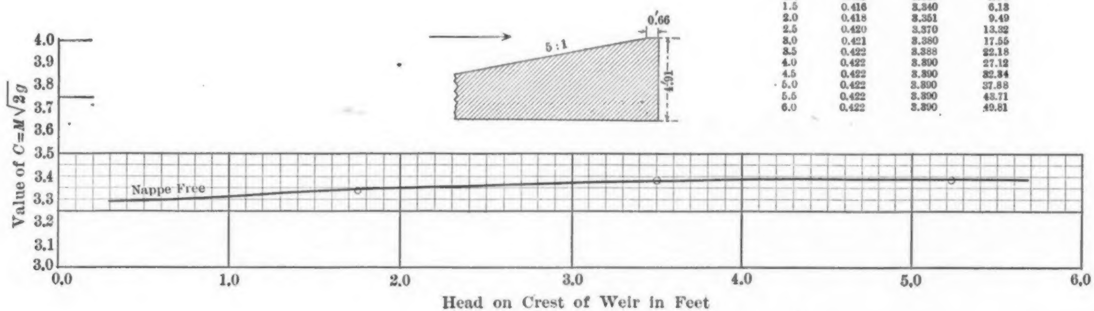
MAY 28TH, 1899.

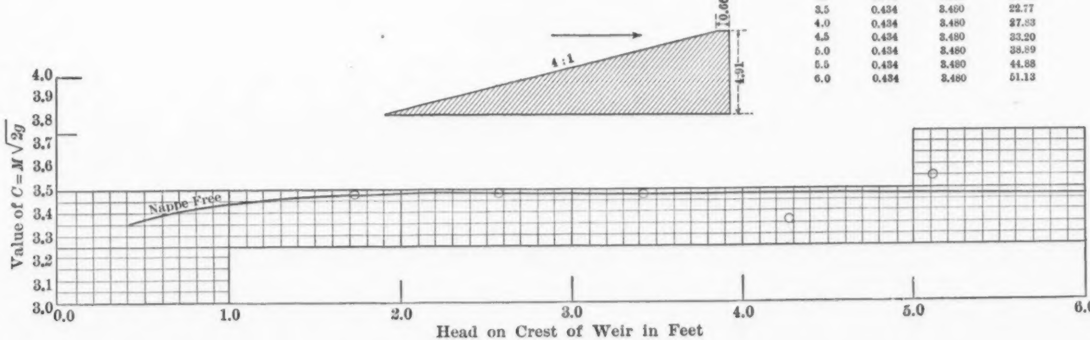
Length of crest 6.58 ft.

Number of experiments, 3.

Limiting heads, 1.75 ft. and 5.23 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per foot.
0.5	0.412	3.310	1.18
1.0	0.415	3.330	3.38
1.5	0.416	3.340	6.13
2.0	0.418	3.351	9.49
2.5	0.420	3.370	13.32
3.0	0.421	3.380	17.55
3.5	0.422	3.388	22.18
4.0	0.422	3.390	27.12
4.5	0.422	3.390	32.34
5.0	0.422	3.390	37.88
5.5	0.422	3.390	43.71
6.0	0.422	3.390	49.81





CORNELL UNIVERSITY

EXPERIMENT No. 4.

MAY 26TH, 1899.

Length of crest 6.58 ft.

Number of experiments, 6.

Limiting heads, 0.917 ft. and 5.11 ft.

Head, in feet. H	M	$C = M\sqrt{2g}$	Q per ft.
0.5			
1.0	0.429	3.440	3.44
1.5	0.432	3.465	6.36
2.0	0.434	3.480	9.83
2.5	0.434	3.485	13.77
3.0	0.434	3.480	18.07
3.5	0.434	3.480	22.77
4.0	0.434	3.480	27.83
4.5	0.434	3.480	33.20
5.0	0.434	3.480	38.89
5.5	0.434	3.480	44.88
6.0	0.434	3.480	51.13

CORNELL UNIVERSITY

EXPERIMENT NO. 5.

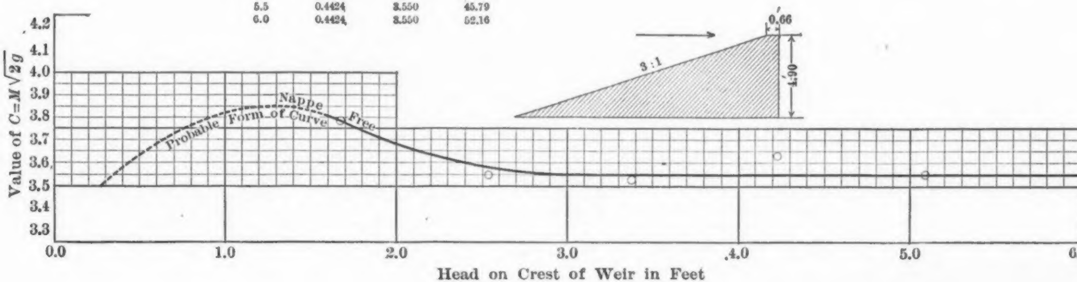
MAY 27TH, 1899.

Length of crest 6.58 ft.

Number of experiments, 5.

Limiting heads, 1.680 ft. and 5.085 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per foot.
0.5	0.454	3.64	1.28
1.0	0.476	3.82	3.82
1.5	0.478	3.83	7.17
2.0	0.4596	3.688	10.42
2.5	0.4474	3.590	14.19
3.0	0.4425	3.551	18.45
3.5	0.4424	3.550	23.24
4.0	0.4424	3.550	28.40
4.5	0.4424	3.550	33.87
5.0	0.4424	3.550	39.68
5.5	0.4424	3.550	45.79
6.0	0.4424	3.550	52.16



CORNELL UNIVERSITY

EXPERIMENT NO. 6.

BAZIN'S SERIES NO. 162.

MAY 27TH, 1899.

Length of crest 6.58 ft.

From full line coefficient curve below

Number of experiments, 5.

Limiting heads, 1.695 ft. and 4.745 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per ft.
0.5	0.5255	4.214	1.50
1.0	0.5288	4.241	4.24
1.5	0.5096	4.088	7.53
2.0	0.4949	3.970	11.22
2.5	0.4848	3.890	15.37
3.0	0.4773	3.830	19.90
3.5	0.4708	3.778	24.75
4.0	0.4661	3.736	29.92
4.5	0.4620	3.708	35.40
5.0	0.4585	3.680	41.14
5.5	0.4548	3.650	47.08
6.0	0.4525	3.630	53.35

CORNELL UNIVERSITY

EXPERIMENT NO. 6.

BAZIN'S SERIES NO. 162.

MAY 27TH, 1899.

Length of crest 6.58 ft.

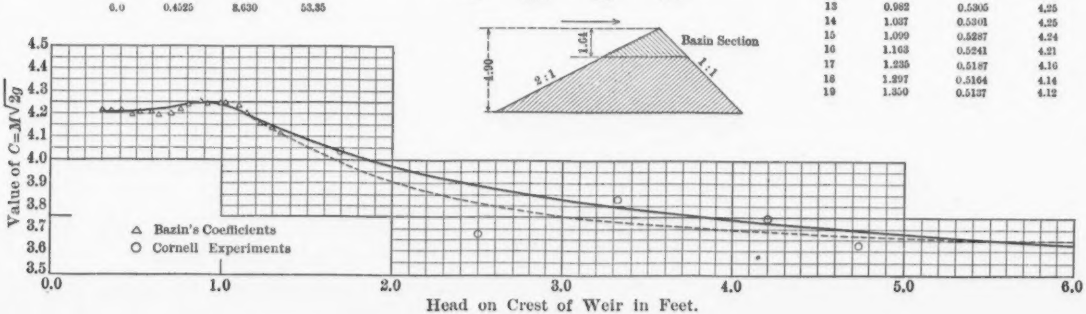
Coefficients from dotted curve on coefficient diagram showing effect of a change of position of the mean coefficient curve.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per ft.
0.5	0.525	4.21	1.50
1.0	0.529	4.24	4.24
1.5	0.505	4.05	7.43
2.0	0.486	3.90	11.02
2.5	0.476	3.82	15.10
3.0	0.467	3.75	19.48
3.5	0.464	3.72	24.35
4.0	0.461	3.70	29.60
4.5	0.459	3.68	35.11
5.0	0.458	3.67	41.03
5.5	0.455	3.65	47.08
6.0	0.453	3.65	53.35

BAZIN'S

SERIES NO. 162.

No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.302	0.5262	4.22
2	0.360	0.5254	4.22
3	0.418	0.5257	4.22
4	0.472	0.5228	4.20
5	0.529	0.5246	4.21
6	0.586	0.5250	4.21
7	0.644	0.5233	4.20
8	0.697	0.5228	4.20
9	0.755	0.5253	4.22
10	0.810	0.5280	4.24
11	0.866	0.5310	4.26
12	0.923	0.5306	4.25
13	0.982	0.5305	4.25
14	1.037	0.5301	4.25
15	1.099	0.5287	4.24
16	1.163	0.5241	4.21
17	1.235	0.5187	4.16
18	1.297	0.5164	4.14
19	1.350	0.5137	4.12



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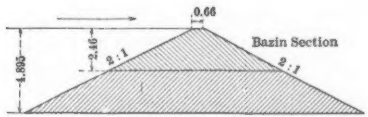


CORNELL UNIVERSITY

EXPERIMENT No. 8.

BAZIN'S SERIES No. 178.

MAY 30TH, 1899.

Length of crest 6.58 ft.

Upstream face covered with ¼" mesh galvanized wire screening.

Number of experiments, 5.

Limiting heads, 1.657 ft. and 5,011 ft.

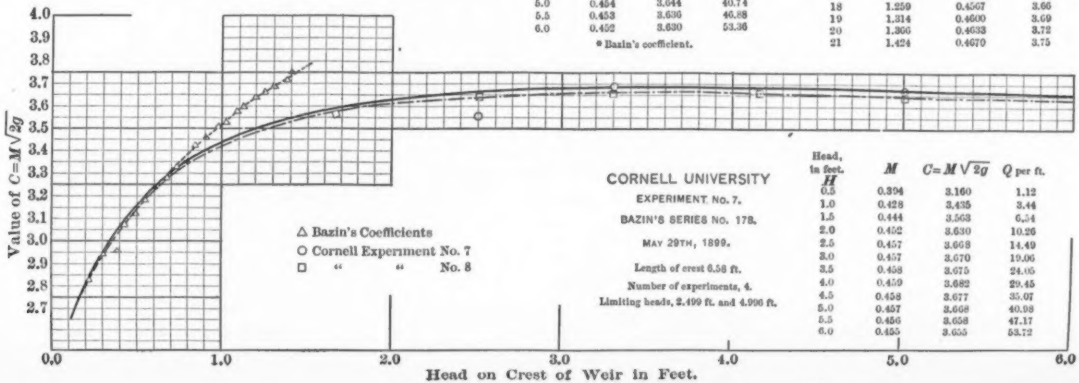
Head, in feet. H	M	$C = M\sqrt{2g}$	Q per ft.
0.5	0.391	3.140 *	1.11
1.0	0.426	3.422	3.42
1.5	0.442	3,548	6.51
2.0	0.450	3.612	10.21
2.5	0.453	3.640	14.39
3.0	0.456	3.658	18.99
3.5	0.457	3.668	24.00
4.0	0.456	3.662	29.30
4.5	0.455	3.653	34.85
5.0	0.454	3.644	40.74
5.5	0.453	3.636	46.88
6.0	0.452	3.630	53.36

* Bazin's coefficient.

BAZIN'S

SERIES No. 178.

No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.222	0.3522	2.83
2	0.299	0.3673	2.95
3	0.367	0.3686	2.96
4	0.431	0.3837	3.08
5	0.491	0.3907	3.13
6	0.556	0.3981	3.19
7	0.614	0.4038	3.24
8	0.669	0.4093	3.28
9	0.732	0.4147	3.33
10	0.789	0.4190	3.36
11	0.847	0.4271	3.43
12	0.906	0.4315	3.46
13	0.966	0.4379	3.51
14	1.028	0.4398	3.53
15	1.083	0.4467	3.58
16	1.142	0.4493	3.60
17	1.195	0.4540	3.64
18	1.259	0.4567	3.66
19	1.314	0.4600	3.69
20	1.366	0.4633	3.72
21	1.424	0.4670	3.75



CORNELL UNIVERSITY

EXPERIMENT No. 7.

BAZIN'S SERIES No. 178.

MAY 29TH, 1899.

Length of crest 6.58 ft.

Number of experiments, 4.

Limiting heads, 2.499 ft. and 4.996 ft.

Head, in feet. H	M	$C = M\sqrt{2g}$	Q per ft.
0.5	0.394	3.160	1.12
1.0	0.428	3.435	3.44
1.5	0.444	3.563	6.54
2.0	0.452	3.630	10.26
2.5	0.457	3.668	14.49
3.0	0.457	3.670	19.06
3.5	0.458	3.675	24.05
4.0	0.459	3.682	29.45
4.5	0.458	3.677	35.07
5.0	0.457	3.668	40.98
5.5	0.456	3.658	47.17
6.0	0.455	3.655	53.72

CORNELL UNIVERSITY

EXPERIMENT NO. 9,

BAZIN'S SERIES NO. 173.

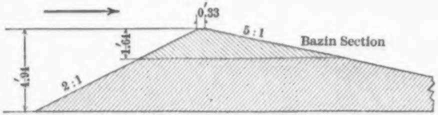
MAY 31ST, 1899.

Length of crest 6.58 ft.

Number of experiments, 4.

Limiting heads, 1.677 ft. and 4.842 ft.

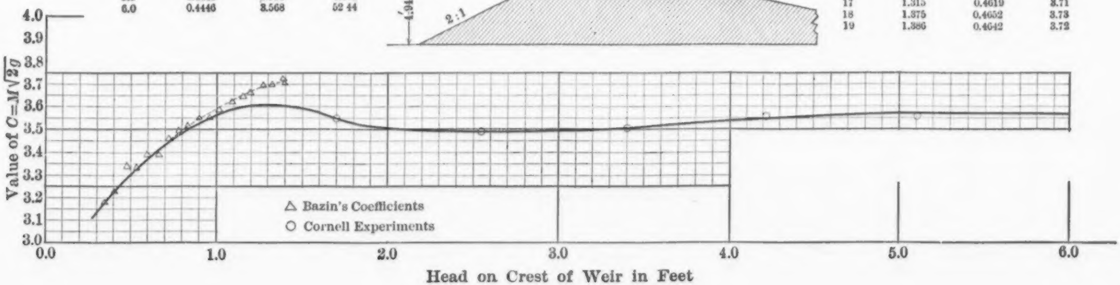
Head, in feet. H	M	$C=M\sqrt{2g}$	Q per foot.
0.5	0.4112	3.300	1.17
1.0	0.4450	3.570	3.57
1.5	0.4480	3.595	6.60
2.0	0.4374	3.510	9.92
2.5	0.4355	3.495	13.83
3.0	0.4356	3.496	18.16
3.5	0.4375	3.510	22.97
4.0	0.4410	3.537	28.30
4.5	0.4436	3.560	33.96
5.0	0.4446	3.568	39.87
5.5	0.4446	3.568	46.01
6.0	0.4446	3.568	52 44



BAZIN'S

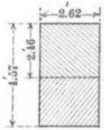
SERIES NO. 173.

No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.346	0.3962	3.18
2	0.410	0.4030	3.23
3	0.471	0.4159	3.34
4	0.532	0.4155	3.33
5	0.594	0.4223	3.39
6	0.658	0.4233	3.39
7	0.714	0.4319	3.46
8	0.769	0.4363	3.50
9	0.834	0.4398	3.52
10	0.892	0.4438	3.56
11	0.960	0.4447	3.56
12	1.021	0.4480	3.59
13	1.074	0.4532	3.63
14	1.137	0.4555	3.65
15	1.196	0.4572	3.67
16	1.260	0.4610	3.70
17	1.315	0.4619	3.71
18	1.375	0.4652	3.73
19	1.386	0.4642	3.72



BAZIN'S
SERIES NO. 114.

No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.204	0.3073	2.47
2	0.280	0.3167	2.54
3	0.352	0.3229	2.59
4	0.433	0.3245	2.60
5	0.504	0.3233	2.59
6	0.578	0.3241	2.60
7	0.657	0.3270	2.62
8	0.735	0.3272	2.63
9	0.810	0.3281	2.63
10	0.882	0.3300	2.65
11	0.958	0.3316	2.66
12	1.034	0.3336	2.68
13	1.112	0.3351	2.69
14	1.171	0.3361	2.70
15	1.243	0.3398	2.73
16	1.301	0.3407	2.73
17	1.384	0.3440	2.76



Bazin's Section

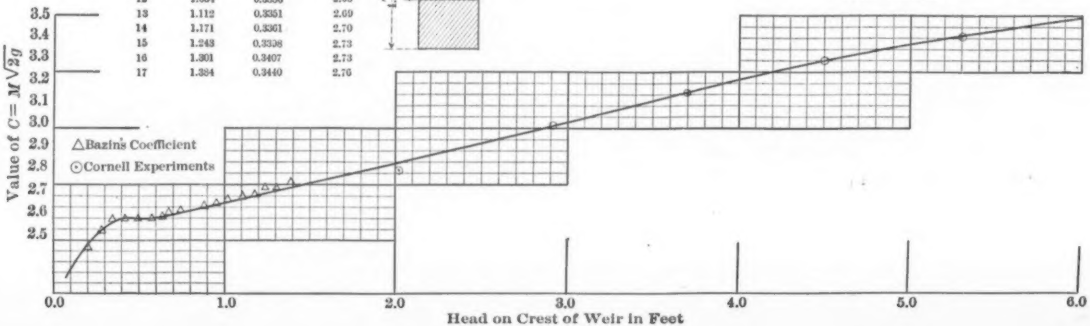
CORNELL UNIVERSITY
EXPERIMENT NO. 10. JUNE 1ST, 1899

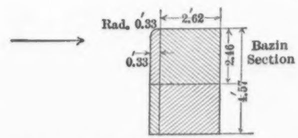
Length of crest 6.56 ft.

Number of experiments, 5.
Limiting heads, 2.038 ft. and 5.324 ft.

Head. In feet. H (1)	M (2)	$C=M\sqrt{2g}$ (3)	Q per ft. (4)
0.5	0.324	2.600*	0.93
1.0	0.333	2.672*	2.67
1.5	0.343	2.750*	5.04
2.0	0.354	2.840	8.02
2.5	0.365	2.928	11.53
3.0	0.376	3.015	15.06
3.5	0.388	3.113	20.37
4.0	0.400	3.208	25.67
4.5	0.411	3.297	31.45
5.0	0.422	3.388	37.86
5.5	0.426	3.420	44.10
6.0	0.433	3.480	51.15

* Bazin's coefficients.





BAZIN'S

SERIES NO. 116.

No. of experiment.	h=observed depth on crest, in feet.	m=coefficient of discharge.	$m\sqrt{2g}$
1	0.177	0.3372	2.71
2	0.225	0.3526	2.83
3	0.296	0.3607	2.90
4	0.367	0.3637	2.92
5	0.435	0.3679	2.95
6	0.504	0.3707	2.98
7	0.537	0.3731	2.99
8	0.639	0.3753	3.01
9	0.713	0.3734	3.00
10	0.781	0.3742	3.00
11	0.849	0.3764	3.02
12	0.917	0.3765	3.02
13	0.986	0.3797	3.05
14	1.053	0.3813	3.06
15	1.120	0.3841	3.08
16	1.185	0.3859	3.09
17	1.251	0.3863	3.10
18	1.317	0.3888	3.12

CORNELL UNIVERSITY

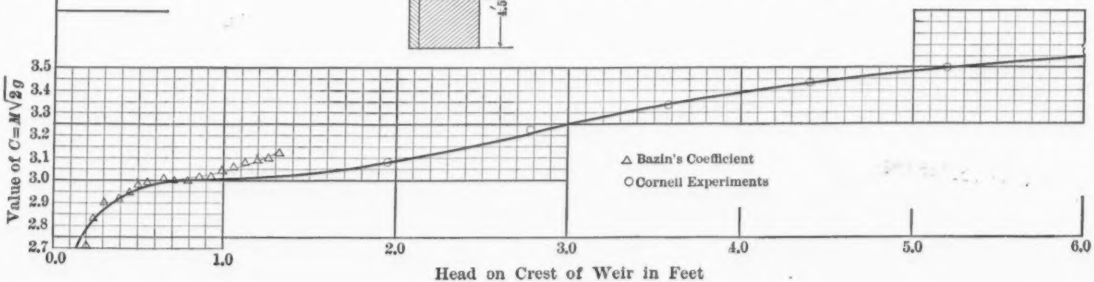
EXPERIMENT NO. 11.

BAZIN'S SERIES NO. 116.

JUNE 1ST, 1899.

Length of crest 6.58 ft.
Number of experiments, 5.
Limiting heads, 1.949 ft. and 5.208 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per foot.
0.5	0.369	2.960	1.06
1.0	0.375	3.008	3.01
1.5	0.378	3.033	5.57
2.0	0.384	3.085	8.72
2.5	0.388	3.115	12.31
3.0	0.405	3.252	16.89
3.5	0.414	3.326	21.77
4.0	0.422	3.385	27.08
4.5	0.428	3.435	32.77
5.0	0.433	3.475	38.84
5.5	0.437	3.510	45.27
6.0	0.442	3.549	52.70



CORNELL UNIVERSITY

EXPERIMENT NO. 12.

BAZIN'S SERIES NO. 115.

JUNE 20, 1899.

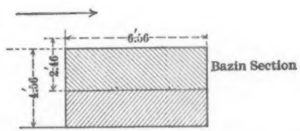
Length of crest 6.58 ft.

Number of experiments, 4.

Limiting heads, 2.175 ft. and 5.102 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per foot.
0.5	0.3117	2.502 *	0.90
1.0	0.3238	2.600	2.60
1.5	0.3170	2.545	4.67
2.0	0.3096	2.485	7.04
2.5	0.3088	2.480	9.80
3.0	0.3133	2.515	13.06
3.5	0.3193	2.563	16.78
4.0	0.3251	2.610	20.88
4.5	0.3307	2.655	25.35
5.0	0.3364	2.700	30.18
5.5	0.3414	2.740	35.33
6.0	0.3437	2.760	40.55

* Bazin's coefficient.

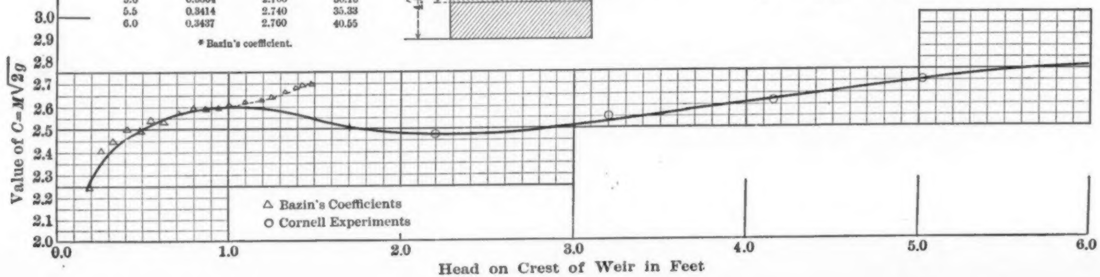


Bazin Section

BAZIN'S

SERIES NO. 115.

No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.196	0.2747	2.25
2	0.264	0.3004	2.41
3	0.342	0.3047	2.45
4	0.415	0.3123	2.51
5	0.495	0.3119	2.50
6	0.566	0.3184	2.55
7	0.638	0.3173	2.54
8	0.716	0.3194	2.56
9	0.792	0.3243	2.60
10	0.871	0.3245	2.60
11	0.948	0.3245	2.60
12	1.023	0.3250	2.61
13	1.097	0.3281	2.63
14	1.178	0.3294	2.64
15	1.260	0.3302	2.65
16	1.330	0.3342	2.68
17	1.388	0.3348	2.69
18	1.424	0.3365	2.70
19	1.467	0.3362	2.70



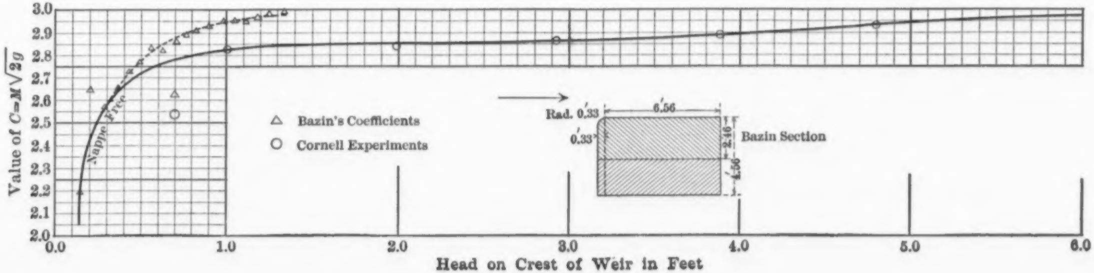
CORNELL UNIVERSITY
EXPERIMENT NO. 13
BAZIN'S SERIES NO. 117
JUNE 20, 1899.

Length of crest 6.58 ft.
Number of experiments, 5.
Limiting heads, 1.016 ft. and 4.880 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per foot.
0.5	0.338	2.716	0.96
1.0	0.352	2.823	2.83
1.5	0.354	2.842	5.22
2.0	0.354	2.844	8.05
2.5	0.355	2.850	11.26
3.0	0.356	2.860	14.85
3.5	0.358	2.877	18.83
4.0	0.360	2.896	23.17
4.5	0.363	2.918	27.84
5.0	0.367	2.945	32.91
5.5	0.369	2.963	38.22
6.0	0.370	2.975	43.70

BAZIN'S
SERIES NO. 117

No. of experiment.	h = observed depth on crest, in feet.	m = coefficient of discharge.	$m\sqrt{2g}$
1	0.158	0.2741	2.19
2	0.204	0.3287	2.64
3	0.289	0.3207	2.57
4	0.361	0.3286	2.65
5	0.426	0.3396	2.73
6	0.494	0.3452	2.77
7	0.562	0.3533	2.83
8	0.635	0.3521	2.82
9	0.708	0.3563	2.86
10	0.771	0.3604	2.89
11	0.834	0 3629	2.91
12	0.912	0.3649	2.93
13	0.989	0 3671	2.95
14	1.064	0.3680	2.95
15	1.129	0.3678	2.95
16	1.197	0.3701	2.97
17	1.267	0.3712	2.98
18	1.336	0.3731	2.99



CORNELL UNIVERSITY

EXPERIMENT NO. 14.

REXFORD FLATS DAM.

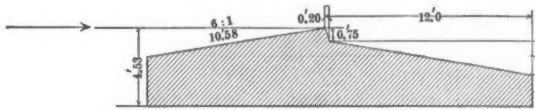
JUNE 3RD, 1899.

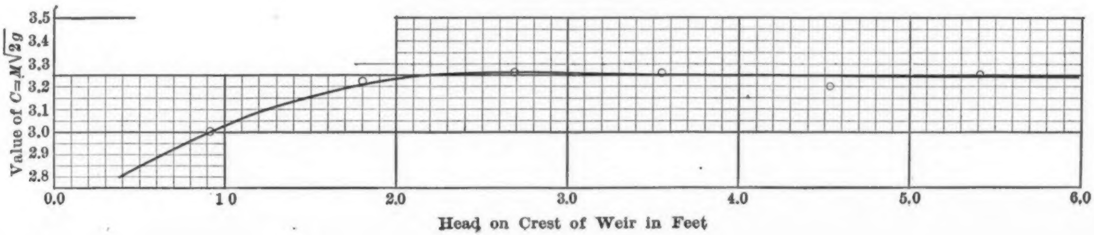
Length of crest 6.58 ft.

Number of experiments, 6.

Limiting heads, 0.924 ft. and 5.418 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per foot.
0.5	0.355	2.850	1.02
1.0	0.377	3.025	3.02
1.5	0.394	3.160	5.80
2.0	0.403	3.232	9.15
2.5	0.406	3.255	12.86
3.0	0.406	3.255	16.90
3.5	0.405	3.250	21.27
4.0	0.405	3.250	26.00
4.5	0.404	3.245	30.97
5.0	0.404	3.242	36.22
5.5	0.403	3.238	41.76
6.0	0.403	3.236	47.56





CORNELL UNIVERSITY

EXPERIMENT No. 15.

REXFORD FLATS DAM.

With up-stream edge of crest rounded to a radius of 0.328 ft.

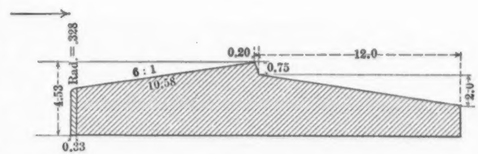
JUNE 30, 1899.

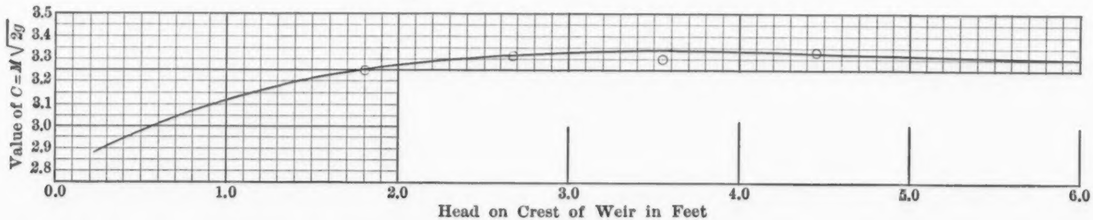
Length of crest 6.58 ft.

Number of experiments, 4.

Limiting heads, 1.786 ft. and 4.439 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per ft.
0.5	0.371	2.980	1.06
1.0	0.389	3.120	3.12
1.5	0.400	3.210	5.90
2.0	0.408	3.270	9.25
2.5	0.412	3.306	13.07
3.0	0.415	3.332	17.32
3.5	0.416	3.340	21.86
4.0	0.415	3.331	26.65
4.5	0.414	3.325	31.72
5.0	0.413	3.315	37.15
5.5	0.412	3.305	42.65
6.0	0.411	3.300	48.48





CORNELL UNIVERSITY

EXPERIMENT No. 16.

LITTLE FALLS DAM, SECTION No. 1.

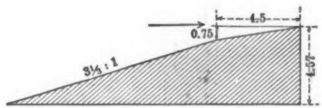
JUNE 5TH, 1899.

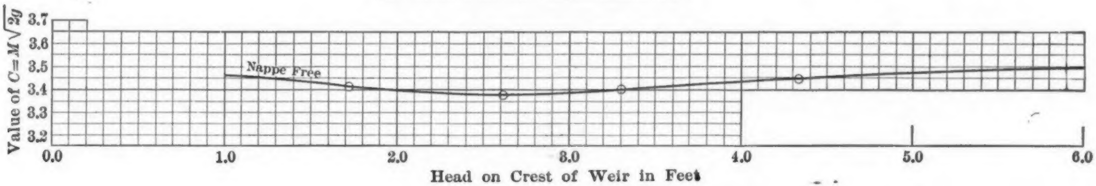
Length of crest 6.58 ft.

Number of experiments, 4.

Limiting heads, 1.735 ft. and 4.339 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per ft.
0.5			
1.0	0.432	3.468	3.47
1.5	0.428	3.434	6.30
2.0	0.424	3.400	9.62
2.5	0.421	3.380	13.36
3.0	0.422	3.385	17.58
3.5	0.425	3.411	22.32
4.0	0.428	3.436	27.48
4.5	0.431	3.460	33.01
5.0	0.433	3.478	38.87
5.5	0.435	3.490	45.00
6.0	0.436	3.500	51.44





CORNELL UNIVERSITY

EXPERIMENT NO. 17

LITTLE FALLS DAM, SECTION NO. 2.

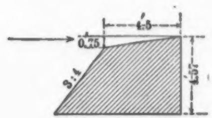
JUNE 5TH, 1899.

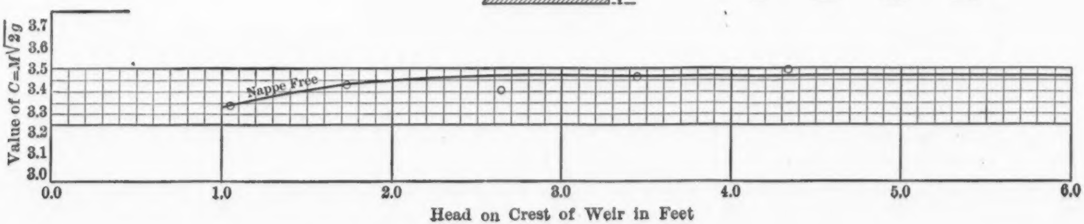
Length of crest 6.58 ft.

Number of experiments, 5.

Limiting heads, 1.067 ft. and 4.336 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per foot.
0.5			
1.0	0.415	3.330	3.33
1.5	0.425	3.403	6.25
2.0	0.430	3.450	9.76
2.5	0.432	3.465	13.70
3.0	0.432	3.470	18.03
3.5	0.432	3.465	22.66
4.0	0.432	3.465	27.73
4.5	0.432	3.465	33.05
5.0	0.432	3.465	38.75
5.5	0.432	3.465	44.08
6.0	0.432	3.465	50.90





CORNELL UNIVERSITY

EXPERIMENT NO. 18.

SECTION OF SPILLWAY OF INDIAN LAKE DAM.

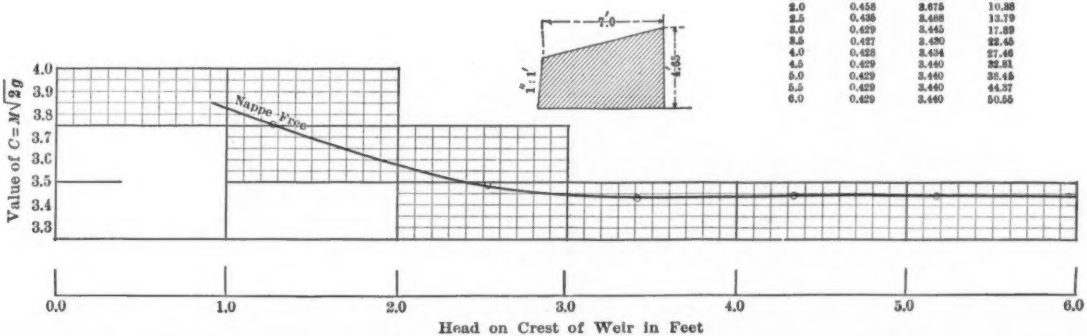
JUNE 6TH, 1899.

Length of crest 6.58 ft.

Number of experiments, 5.

Limiting heads, 1.277 ft. and 5.180 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per foot.
0.5			
1.0	0.476	3.820	3.82
1.5	0.46[illegible]	3.695	6.78
2.0	0.458	3.675	10.88
2.5	0.435	3.488	13.79
3.0	0.429	3.445	17.89
3.5	0.427	3.430	22.45
4.0	0.428	3.434	27.46
4.5	0.429	3.440	32.81
5.0	0.429	3.440	38.45
5.5	0.429	3.440	44.37
6.0	0.429	3.440	50.55



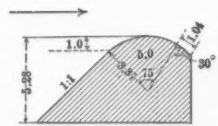
CORNELL UNIVERSITY
EXPERIMENT No. 19.
SECTION FOR SUBMERGED WEIR.
JUNE 7TH, 1899.

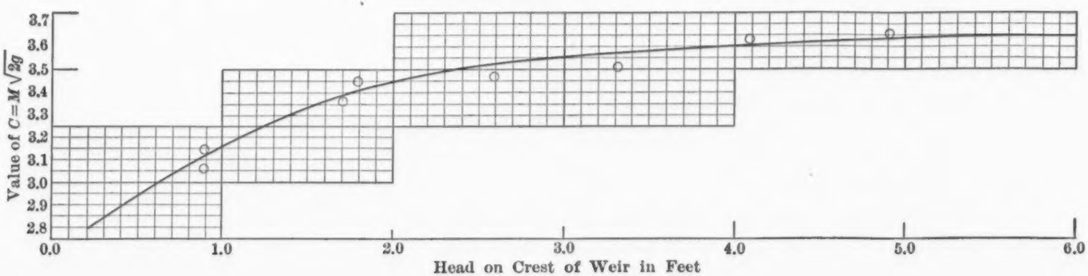
Length of crest 6.58 ft.

Number of experiments, 8

Limiting heads, 0.887 and 4.913 ft.

Head, in feet. H	M	$C=M\sqrt{2g}$	Q per ft.
0.5	0.367	2.945	1.04
1.0	0.394	3.165	3.17
1.5	0.412	3.305	6.06
2.0	0.430	3.454	9.77
2.5	0.439	3.520	13.91
3.0	0.443	3.558	18.48
3.5	0.447	3.585	23.46
4.0	0.450	3.610	28.88
4.5	0.452	3.630	34.63
5.0	0.455	3.650	40.80
5.5	0.457	3.665	47.28
6.0	0.458	3.675	54.02





On April 14th, 1899, the writer visited the new Cornell University Hydraulic Laboratory and at once saw that a fine opportunity was offered there to experiment on flows over weirs at much higher heads than had hitherto been possible; and, on communicating his views to the United States Board of Engineers on Deep Waterways, he was permitted to undertake a series of experiments in co-operation with the University authorities. Messrs. Wallace Greenalch, Assoc. M. Am. Soc. C. E., Robert E. Horton, and George E. Cook were detailed from the Deep Waterways engineering corps for this work, which was done in co-operation with Professor Gardner S. Williams, M. Am. Soc. C. E., Engineer in Charge of the Hydraulic Laboratory, permission to use the same having been obtained by correspondence with Professor E. A. Fuertes, M. Am. Soc. C. E., Director of Cornell University College of Civil Engineering. The writer gave the experiments general supervision, the working out of the details being mostly left to Mr. Greenalch and Professor Williams, Mr. Greenalch undertaking to construct the necessary flumes, bulkheads, experimental weirs, etc., and Professor Williams preparing and taking charge of the measuring apparatus. The reductions were made by Messrs. Greenalch, Horton and Cook under the direction of the writer.

Fig. 1 is a plan and section of the experimental channel at the Cornell University Hydraulic Laboratory. This laboratory has been quite fully described in *Engineering News* for March 2d, 1899, and no farther description will be given here than is necessary to explain the experiments.

Description of the Arrangements for the Experiments.—The canal in which the experiments were made consists, briefly, of a channel with sides and bottom of concrete. It is 418 ft. long, 16 ft. wide and 10 ft. deep. The gradient of the bottom of the channel is at the rate of 1 ft. in 500. A bulkhead composed of 12 x 12-in. timbers, situated about 60 ft. from the upper end, divides the channel into two chambers. A standard sharp-edged weir, 16 ft. in length, was placed on this bulkhead, the crest of the weir being 13.13 ft. above the bottom of the channel. The upper chamber above the bulkhead has higher side walls than the lower chamber, which admitted of a depth of 17.7 ft. of water. At the lower end of the channel another timber bulkhead closes the lower chamber, and on this the weirs to be experimented upon were built. The top of this bulkhead was about 4.8 ft. above the

bottom of the channel. The heights vary slightly for each experimental weir, the exact height of each being shown on the sections at the head of the tabulations of results.

In order to obtain heads of about 5 ft. on the lower weir the 16-ft. channel was narrowed to a width of 6.56 ft. (2 m.) by means of a wooden flume, as shown in Fig. 1. This flume was 6.56 ft. wide for a distance of 48 ft. above the lower bulkhead and then expanded to a width of 16 ft. in a length of 8.3 ft., as shown. The flume was constructed of matched, white pine boards 1.75 ins. thick, planed on the inside and held in place by bents of 4 x 4-in. timbers. As the lower bulkhead was water-tight for the whole width of the channel, no attempt was made to construct the sides and bottoms of the flume absolutely water-tight, although they were practically so. Inasmuch as nearly equal pressure on both sides of flume would permit of greater economy in construction, two boards were left off each side at the upper end, thus allowing the water to enter at the sides between the flume and the concrete walls of the main canal. This arrangement also diminished greatly the area of water-tight work. The sides of the flume were extended from 8 to 24 ft. below the bulkhead, according to the form of weir experimented upon, thus preventing lateral expansion of the nappe after passing the weir and crest. Openings were left in each side of this extension below the level of the crest in order to certainly allow free access of air under the nappe. The vertical fall of the water from the crest of the experimental weir to the rock below was about 12.2 ft.

There is a pond of 22 acres above the main reservoir dam, the surface of which was raised about 1.7 ft. by means of flash boards placed on the main spillway. The water surface thus obtained was about 5.4 ft. above the crest of the standard weir. Water from the reservoir was admitted into the upper chamber through six wooden sluice gates, operated by rack and pinion apparatus with long levers.

The sharp-edged standard weir was composed of a 3.5 x 5-in. steel angle, secured by lag screws to a 6 x 12-in. oak timber, as shown in Fig. 1. After bolting the angle-iron into place on the timber, the edge of the 5-in. leg was planed and dressed to a true line $\frac{5}{16}$ in. in width, and carefully leveled in position on the upper bulkhead. Air was admitted freely under the nappe by means of deflecting boards at each end of the weir. The fall of the water from the crest of the standard

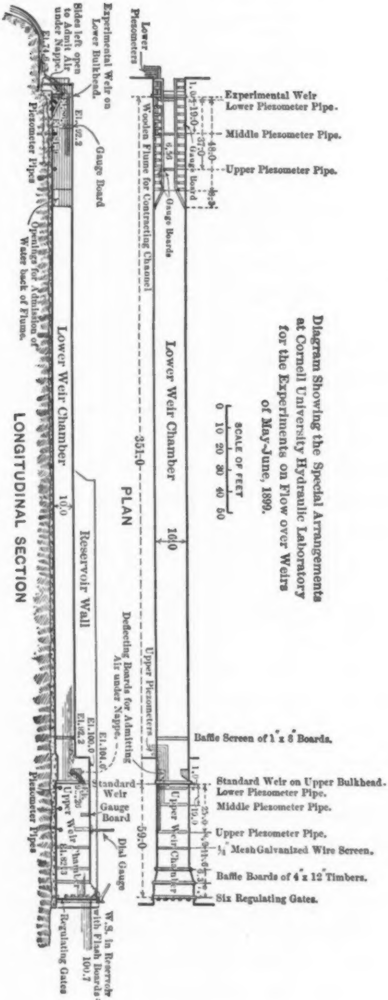


Diagram Showing the Special Arrangements
at Cornell University Hydraulic Laboratory
for the Experiments on Flow over Weirs
of May-June, 1899.
SCALE OF FEET
0 10 20 30 40 50
Experimental Weir
Lower Piezometer Pipe.
Middle Piezometer Pipe.
Upper Piezometer Pipe.
Lower Piezometers
Wooden Flume for Contracting Channel
Gauge Boards
Gauge Board
Lower Weir Chamber
351.0
PLAN
Upper Piezometers
Deflecting Boards for Admitting Air under Nappe.
Baffle Screen of 1" x 8" Boards.
Standard Weir on Upper Bulkhead.
Lower Piezometer Pipe.
Middle Piezometer Pipe.
Upper Piezometer Pipe.
¼" Mesh Galvanized Wire Screen.
Baffle Boards of 4" x 12" Timbers.
Six Regulating Gates.
Upper Weir Chamber
69.0
Experimental Weir on Lower Bulkhead.
Sides left open to Admit Air under Nappe
Gauge Board
Piezometer Pipes
Openings for Admission of Water back of Flume.
Lower Weir Chamber
Reservoir Wall
Standard Weir Gauge Board
Dial Gauge
W.S. in Reservoir with Flash Boards = 100.7
Piezometer Pipes
Regulating Gates
LONGITUDINAL SECTION

FIG. 1.

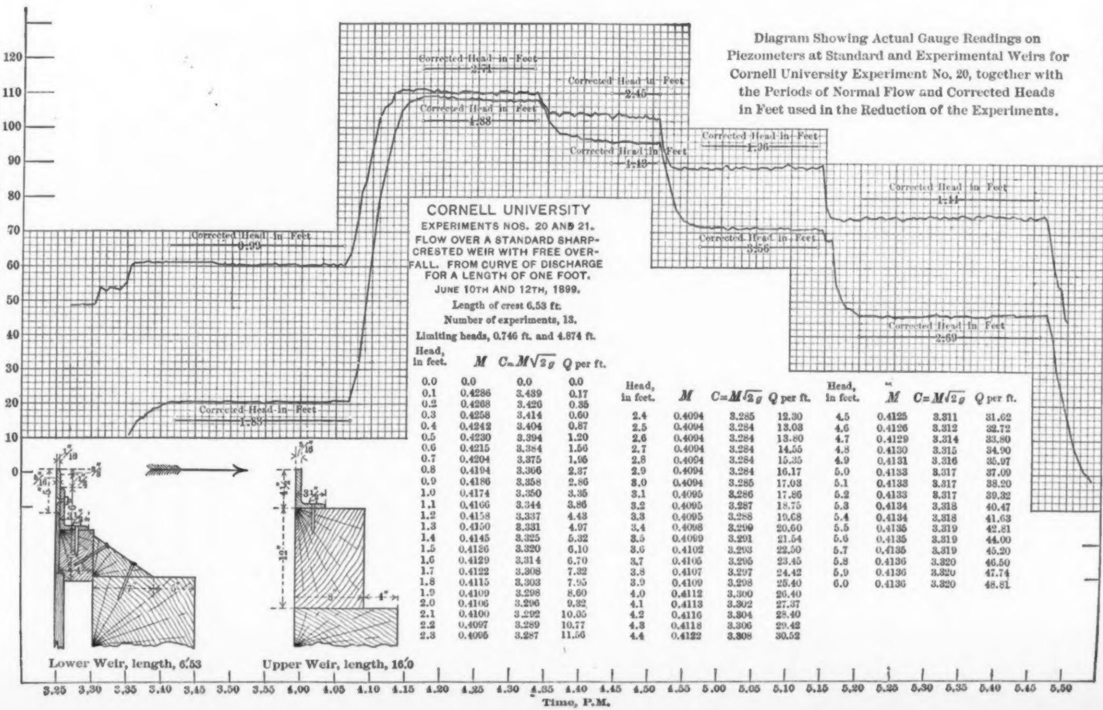
weir to the water surface in the lower chamber varied from about 3 to 8 ft., according to the quantity of water flowing.

The velocity of the water passing through the regulating gates at the extreme head of the channel was checked by three screens in the upper chamber. The first two consisted of 4 x 12-in. timbers placed horizontally with the wide face toward the current, and spaced from 8 to 12 ins. apart. Below these was a third screen of $\frac{1}{4}$ in. mesh, galvanized wire netting. A screen, composed of 1 x 8 in. boards, laid horizontally with the edges to the current and spaced 2 ins. apart, was placed about 20 ft. below the upper bulkhead, and served to quiet the water in the lower chamber.

The heads on the weirs were measured by means of piezometers, constructed as follows: A 1-in. galvanized iron pipe, with holes $\frac{1}{4}$ in. in diameter and spaced 6 ins. apart, was laid across the channel about 8 ins. above the bottom, with the holes therein opening downward. Connections with these pipes were made by $\frac{3}{4}$-in. pipes passing through the bulkhead to a point below the weir, where the gauges could easily be connected by rubber hose. The gauges were glass tubes, $\frac{3}{4}$ in. internal diameter, mounted on wooden standards, and read by a scale graduated to 2-mm. spaces. Three piezometers were set at each weir, as shown in Fig. 1, though readings were taken only on the upper two. In order to check the accuracy of the piezometric readings, at the conclusion of Experiment No. 17, a fourth piezometer pipe was set in the bottom of the flume above the lower bulkhead, and about 6 ins. up stream from the upper piezometer. This pipe was set with $\frac{1}{4}$-in. holes directly on top, and with the top of the pipe flush with the bottom of the flume. Readings were taken simultaneously on both piezometers, and considerable differences noted.

At the lower weir the height of the flowing water in the flume and of the still water behind the flume was read on scales marked on the side of the flume. These scales were divided to 0.05 ft. and read by interpolation to 0.01 ft. Similar gauges were set in the upper chamber and in the reservoir, and readings of each were taken every five minutes.

In addition to the gauge board in the upper chamber, a float gauge, read by dial to 0.01 ft., was set directly over the upper piezometer. The float of this dial gauge was a heavy, sheet-tin, air-tight vessel, weighted with shot and caused to move vertically with the water in the interior of a length of 8-in. cast-iron pipe, suspended from two timbers



CORNELL UNIVERSITY
EXPERIMENTS NOS. 20 AND 21.
FLOW OVER A STANDARD SHARP-CRESTED WEIR WITH FREE OVERFALL. FROM CURVE OF DISCHARGE FOR A LENGTH OF ONE FOOT.
JUNE 10TH AND 12TH, 1899.
Length of crest 6.53 ft.
Number of experiments, 13.
Limiting heads, 0.746 ft. and 4.874 ft.

Head, in feet.	M	$C=M\sqrt{2g}$	Q per ft.	Head, in feet.	M	$C=M\sqrt{2g}$	Q per ft.	Head, in feet.	M	$C=M\sqrt{2g}$	Q per ft.
0.0	0.0	0.0	0.0								
0.1	0.4286	3.439	0.17								
0.2	0.4268	3.426	0.35								
0.3	0.4258	3.414	0.60	2.4	0.4094	3.285	12.30	4.5	0.4125	3.311	31.62
0.4	0.4242	3.404	0.87	2.5	0.4094	3.284	13.03	4.6	0.4126	3.312	32.72
0.5	0.4230	3.394	1.20	2.6	0.4094	3.284	13.80	4.7	0.4129	3.314	33.80
0.6	0.4215	3.384	1.56	2.7	0.4094	3.284	14.55	4.8	0.4130	3.315	34.90
0.7	0.4204	3.375	1.96	2.8	0.4094	3.284	15.35	4.9	0.4131	3.316	35.97
0.8	0.4194	3.366	2.37	2.9	0.4094	3.284	16.17	5.0	0.4133	3.317	37.09
0.9	0.4186	3.358	2.86	3.0	0.4094	3.285	17.03	5.1	0.4133	3.317	38.20
1.0	0.4174	3.350	3.35	3.1	0.4095	3.286	17.86	5.2	0.4133	3.317	39.32
1.1	0.4166	3.344	3.86	3.2	0.4095	3.287	18.75	5.3	0.4134	3.318	40.47
1.2	0.4158	3.337	4.43	3.3	0.4095	3.288	19.68	5.4	0.4134	3.318	41.63
1.3	0.4150	3.331	4.97	3.4	0.4098	3.290	20.60	5.5	0.4135	3.319	42.81
1.4	0.4145	3.325	5.32	3.5	0.4099	3.291	21.54	5.6	0.4135	3.319	44.00
1.5	0.4136	3.320	6.10	3.6	0.4102	3.293	22.50	5.7	0.4135	3.319	45.20
1.6	0.4129	3.314	6.70	3.7	0.4105	3.295	23.45	5.8	0.4136	3.320	46.50
1.7	0.4122	3.308	7.32	3.8	0.4107	3.297	24.42	5.9	0.4136	3.320	47.74
1.8	0.4115	3.303	7.95	3.9	0.4109	3.298	25.40	6.0	0.4136	3.320	48.81
1.9	0.4109	3.298	8.60	4.0	0.4112	3.300	26.40				
2.0	0.4106	3.296	9.32	4.1	0.4113	3.302	27.37				
2.1	0.4100	3.292	10.05	4.2	0.4116	3.304	28.40				
2.2	0.4097	3.289	10.77	4.3	0.4118	3.306	29.42				
2.3	0.4096	3.287	11.56	4.4	0.4122	3.308	30.52				

FIG. 2.

across the upper channel. The dial was placed in such a position as to be read easily by the assistants operating the gates at the upper end of the channel, thus insuring that the depth over the standard weir be easily maintained at substantially a constant head, as required.

The experimental weirs of different sections were all made of planed, white pine boards about 1 in. thick, so fastened together with screws as to be easily removable. Leakage from the bulkheads and gates at the lower end of the channel was effectively stopped by caulking with oakum and pitch, and also by the application of wheat bran from time to time.

Observations showed that the upper bulkhead was practically water-tight, but at the side gates, near the lower bulkhead, there was a slight leakage, which has been taken at 0.5 cu. ft. per second for high heads and 0.25 cu. ft. per second for low heads, and proportionally between. The main channel, with side walls of concrete, is considered to be water-tight.

The foregoing allowances for leakage are taken to cover the slight evaporation and absorption loss into the sides of the main channel.

Reduction of the Experiments.—The method of conducting the experiments was, in general, as follows: The main head gates at the entrance to the canal were open to such a width that the dial gauge showed a head of 3 ft. above the standard 16-ft weir. They were retained in this position until a uniform regimen of flow was established in the canal and maintained for a period of from 10 to 20 minutes, during which time the piezometers were read at both weirs at intervals of 30 seconds. At the close of such a period the head gates were lowered until the dial gauge showed a reading of 2.5 ft. head on the upper weir, sufficient time being allowed to elapse to establish and maintain a new regimen of flow, the same as before. In this way piezometric observations were taken during several periods at different heads in each experiment, usually terminating with a head of about 6 ins. or 1 ft. on the upper weir. In some cases the varying of the head on the upper weir by uniform decrements of 6 ins. was not adhered to.

The method of treating the piezometric readings, obtained as described in the preceding paragraph, is shown in Figs. 2 and 3. In Fig. 2 the upper curve shows the readings taken at the upper weir (16 ft. in length), and the lower curve the readings taken at the lower experimental weir for Experiment No. 14 on the Rexford Flats dam. These

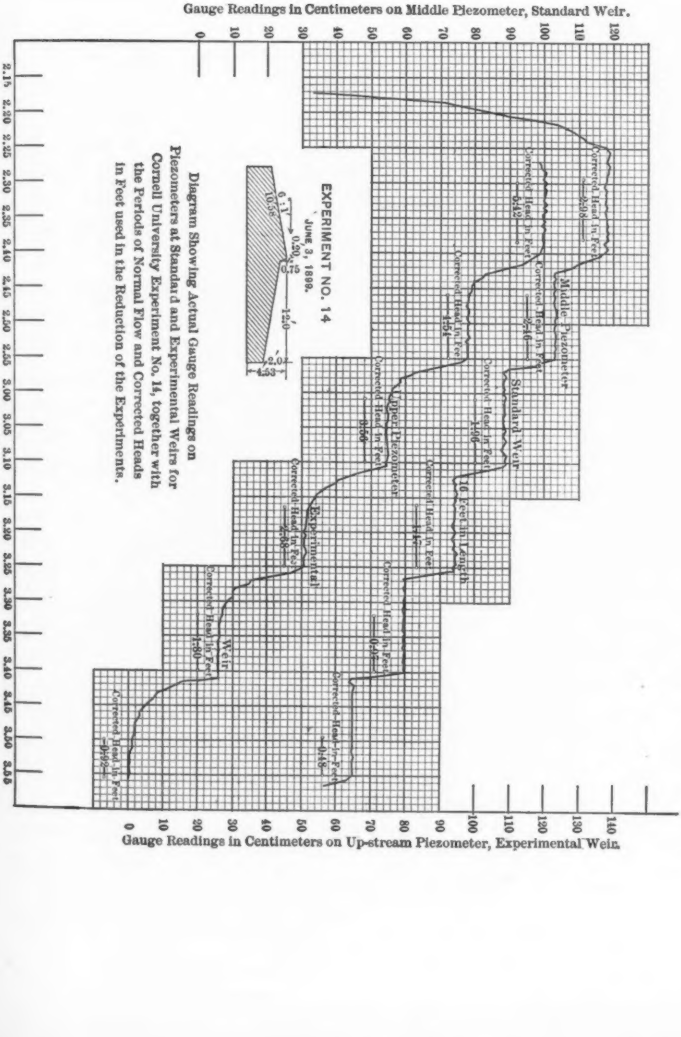


Diagram Showing Actual Gauge Readings on Piezometers at Standard and Experimental Weirs for Cornell University Experiment No. 14, together with the Periods of Normal Flow and Corrected Heads in Feet used in the Reduction of the Experiments.

FIG 8.

two curves, as plotted, show the actual readings taken, in centimeters, without corrections of any sort or kind. The several periods (A-A) of the upper curve, and the corresponding periods (B-B) of the lower curve, represent the actual periods taken in the reductions for each height experimented upon. A mean of the actual readings for these periods has been taken as the mean head during each experimental period.

Fig. 3 is a plotting of similar curves for Experiment No. 20 on a sharp-crested weir, made on June 10th, 1899. The explanations for Fig. 2 apply equally to this figure. Attention may be directed to the method of exhibiting the continuous curve of flow, as shown in Figs. 2 and 3. Its use in the present case is to be credited to Professor Williams. The writer has never seen it used before, and, if it is original with Professor Williams, he is entitled to very great credit for this particular feature of the experiments.

In order to calibrate the upper weir (16 ft. in length), and thereby determine the quantity of water flowing in the lower canal and over the experimental weirs, Experiments Nos. 20 and 21 were made. These experiments apply to a sharp-crested weir of standard form, 5.26 ft. in height, placed in the lower end of the canal. As a basis for the reduction of Experiments Nos. 20 and 21 a discharge curve has been computed for the upper weir for heads up to 0.6 m. (1.969 ft.), using Bazin's formula:

$$Q = M\,L\,H\sqrt{2\,g\,H}$$

$$M = n\left[1 + 0.55\left(\frac{H}{p+h}\right)^2\right]$$

$$m = n\left[1 + 0.55\left(\frac{h}{p+h}\right)^2\right]$$

in which

Q = discharge over weir, in cubic feet per second;
h = observed head on crest, in feet;
h_v = correction for velocity of approach;
H = head on weir corrected for velocity of approach $= h + h_v$;
m = coefficient of discharge in the formula $m\,l\,h\sqrt{2\,g\,h}$;
M = coefficient of discharge in the formula $M\,L\,H\sqrt{2\,g\,H}$;
C = coefficient in the Francis formula $C = M\sqrt{2\,g}$, when g is expressed in feet;
p = height of crest of weir above bottom of channel of approach, in feet;
n = a coefficient which depends on p and h, and which has been taken from Bazin's table.*

* See *Annales des Ponts et Chaussées*, 1888, p. 446.

In the reduction of Experiments Nos. 20 and 21, the mean readings of the middle piezometer at the upper weir were added to the difference of elevation of the gauge zero and of the mean crest as a basis for computing the flow in the upper channel. The discharge corresponding to the heads so obtained was then taken off the discharge curve computed from Bazin's formula for all the periods in which the observed head was less than 0.6 m. (1.969 ft.). The approximate correction to be applied for velocity of approach to the standard weir was then computed as follows :

Let H = the true head on crest of weir, in feet;
h = the observed head, in feet ;
Q_h = discharge over the weir under the head h per lineal foot of crest ;
Q = discharge under the head H per lineal foot of crest ; and
p = height of weir crest above channel bottom, in feet. Then

velocity of approach $= v = \sqrt{2g(H-h)}$,
also,

$$v = \frac{Q}{p+h} = \frac{Q}{A} \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$$\text{and } h_v = (H-h) = \frac{v^2}{2g} = \left(\frac{Q}{A}\right)^2 \times \frac{1}{2g} \dots\dots\dots\dots (2)$$

Q_h being determined from the discharge curve, an approximate value of v and of the corresponding velocity head was computed and the approximate value of the velocity head so obtained added to the observed head h, which was used in determining Q, v and $(H-h)$ with more precision. Generally, two successive applications of these formulas were found sufficient to determine the velocity head with the desired degree of accuracy. In this way the final corrected head $H = h + h_v$ was obtained. After applying a correction for leakage, percolation and surface evaporation, the corresponding discharges by Bazin's formula have been used in determining the coefficients for the sharp-crested experimental weir (6.53 ft. in length) for heads up to 3.5 ft., as produced by heads not exceeding 2 ft. on the upper weir (16 ft. in length).

The foregoing correction for velocity of approach is merely the addition to the observed heads of $\frac{v^2}{2g}$, as determined for the actual flows of each experiment. Messrs. Fteley and Stearns have experi-

mented on the effect of velocity of approach, especially with reference to that part of it represented by the *vis viva* of the water, and state in their classical paper * that, for the conditions of their experiments, corrections of velocity of approach to be added to the observed heads are best represented by

$$1.45 \text{ to } 1.50 \times \left(\frac{v^2}{2g}\right).$$

The problem of correction for velocity of approach is discussed at length by Hamilton Smith, Jr., M. Am. Soc. C. E., in his "Hydraulics," the conclusion being that, for a weir with full contraction and having an unobstructed channel of considerable length, the correction should be about

$$1.1 \text{ to } 1.25 \times \left(\frac{v^2}{2g}\right).$$

For end contractions suppressed, he adopts the values

$$1.33\left(\frac{v^2}{2g}\right) \text{ and } 1.40\left(\frac{v^2}{2g}\right).$$

In the present case it has seemed preferable to use

$$h_v = \frac{v^2}{2g},$$

although the entire suppression of end contractions might appear to indicate a higher correction for the velocity of approach. This view, however, is based upon other considerations, namely, the actual locations of the piezometers.

Messrs. Fteley and Stearns have pointed out that for standard sharp-crested weirs the head should be measured about 6 ft. back from the crest, but in the present case the heads have been measured much farther back. At the upper 16-ft. weir the heads were measured at the middle piezometer for all experiments, except Nos. 1, 2, 3 and 4, for which they were measured at the upper piezometer. These latter observations have been reduced to the basis of the middle piezometer—which was 10 ft. back from the weir—by methods to be detailed farther on.

At the lower or experimental weirs the heads were measured for all experiments at the upper piezometer 38 ft. above the bulkhead on which the experimental weirs were placed. This location was selected in order to insure the piezometers being well above the long back

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* *Transactions*, Am. Soc. C. E., Vol. xii, p. 1.

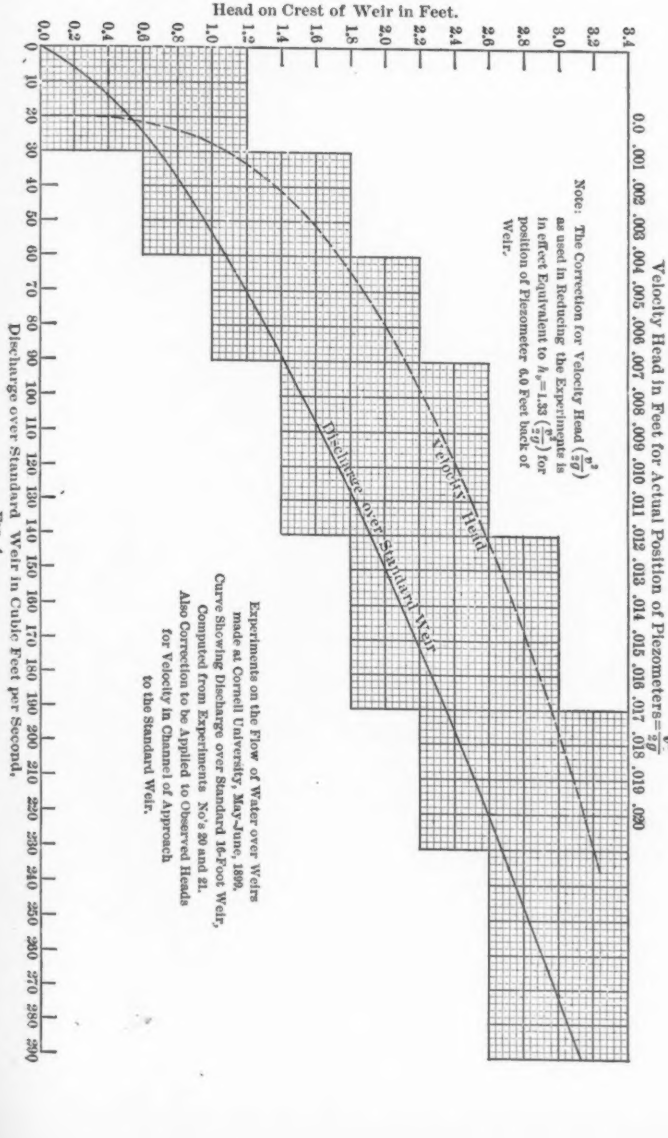
Head on Crest of Weir in Feet.
Velocity Head in Feet for Actual Position of Piezometers $=\frac{v^2}{2g}$
Note: The Correction for Velocity Head $\left(\frac{v^2}{2g}\right)$ as used in Reducing the Experiments is in effect Equivalent to $h_v=1.33\left(\frac{v^2}{2g}\right)$ for position of Piezometer 6.0 Feet back of Weir.
Velocity Head
Discharge over Standard Weir
Experiments on the Flow of Water over Weirs made at Cornell University, May-June, 1899. Curve Showing Discharge over Standard 16-Foot Weir, Computed from Experiments No's 20 and 21. Also Correction to be Applied to Observed Heads for Velocity in Channel of Approach to the Standard Weir.
Discharge over Standard Weir in Cubic Feet per Second.

FIG. 4.

slopes of Experiments Nos. 2, 3, 14, 15 and 16, or any other similar series which it might appear desirable to make.

Messrs. Fteley and Stearns remark that the only inaccuracy to come from measuring the heads more than 6 ft. back will be that due to surface slope. We will now examine as to the possible effect of this in the present case.

For an observed head of 2.693 ft. on the standard 16-ft. weir

$$\frac{v^2}{2g} = 0.013 \text{ ft.} \qquad (a)$$

and

$$1.33\left(\frac{v^2}{2g}\right) = 0.017 \text{ ft.} \qquad (b)$$

The difference of 0.004 ft. is far enough within the limit of accuracy to be negligible.

The corresponding observed head on the experimental weir, 6.56 ft. in length, is 4.677 ft., giving

$$\frac{v^2}{2g} = 0.198 \text{ ft.} \qquad (c)$$

also

$$1.33\left(\frac{v^2}{2g}\right) = 0.270 \text{ ft.} \qquad (d)$$

The difference is 0.072 ft.

Taking the formula $v = C\sqrt{r\,s}$, in which $r = \frac{A}{P}$ and $s = \frac{h}{l}$, and with l equal to $(38 - 6) = 32$ ft.—the distance between where the piezometer should have been to comply with theoretical conditions for a standard weir and where it was actually set—and computing for h_s under a head of 4.677 ft. on the experimental sharp-crested weir, we find $h_s = 0.061$ ft., which differs from the preceding difference of 0.072 ft. by 0.011 ft. At slightly lower heads this difference disappears so rapidly as to become inappreciable, so far as effect on the coefficients of discharge is concerned.

It was concluded, therefore, that for the conditions of the present case,

$$h_v = \frac{v^2}{2g}$$

gave more nearly the true correction than any other accepted formula.

In his experiments comparing flows over standard weirs with flows over the several experimental sections, Bazin himself did not make

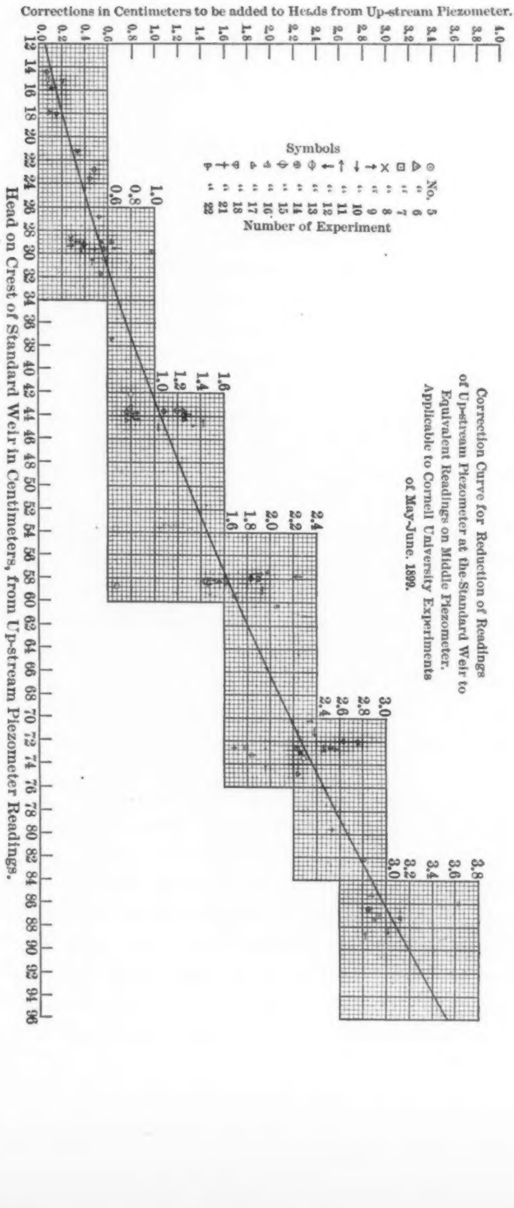
Corrections in Centimeters to be added to Heads from Up-stream Piezometer.
Symbols
Number of Experiment
Correction Curve for Reduction of Readings
of Up-stream Piezometer at the Standard Weir to
Equivalent Readings on Middle Piezometer.
Applicable to Cornell University Experiments
of May-June. 1899.
Head on Crest of Standard Weir in Centimeters, from Up-stream Piezometer Readings.

FIG. 5.

any corrections for velocity of approach. By working on the ratio $\frac{m}{m'}$ the necessity for such corrections was substantially eliminated. On this point, note what he says on page 259.

Final coefficients of discharge for the several weirs have been obtained from the formula

$$C = M\sqrt{2g} = \frac{Q}{LH^{\frac{3}{2}}}$$

in which

Q = total flow over experimental weir, in cubic feet per second;

L = length of crest of experimental weir, in feet; and

H = final corrected head on experimental weir, in feet. Q having been previously found, H was determined from the mean of the readings, for each period of experiment, of a piezometer connected to a horizontal tube placed flush with the bottom of the channel of approach. The correction for velocity of approach was then computed by the formulas just given, using appropriate values of p and h, Q being known from the previous work, as described.

The values of the coefficients M and C, connected by the relation

$$C = M\sqrt{2g},$$

having been determined for the actual heads deduced from the experiments, the values of C so obtained were plotted and a mean curve drawn to best represent the observations. A new series of coefficients, advancing by equal increments of H, have been read from the curve so obtained, as shown by the tabulations of the experiments, Nos. 1 to 21, on pages 267 to 284, inclusive, and in Fig. 2.

Values of Q, the discharge per lineal foot of crest, have been computed by the formula

$$Q = CH^{\frac{3}{2}}.$$

Having obtained the discharge over the sharp-crested experimental weir for heads up to 3.5 ft., the curve of discharge for the standard weir was extended to the height of 3 ft. by using the coefficients obtained as just described. Fig. 4 shows this curve, as well as the correction curve to be applied to observed heads for velocity in the channel of approach to the standard weir. The additional experiments made on a sharp-crested weir at heads above 3.5 ft. were then reduced in precisely the same manner as before, giving finally a series of coefficients of discharge over a sharp-crested weir with a range of heads from 0.746 ft. to 4.874 ft.

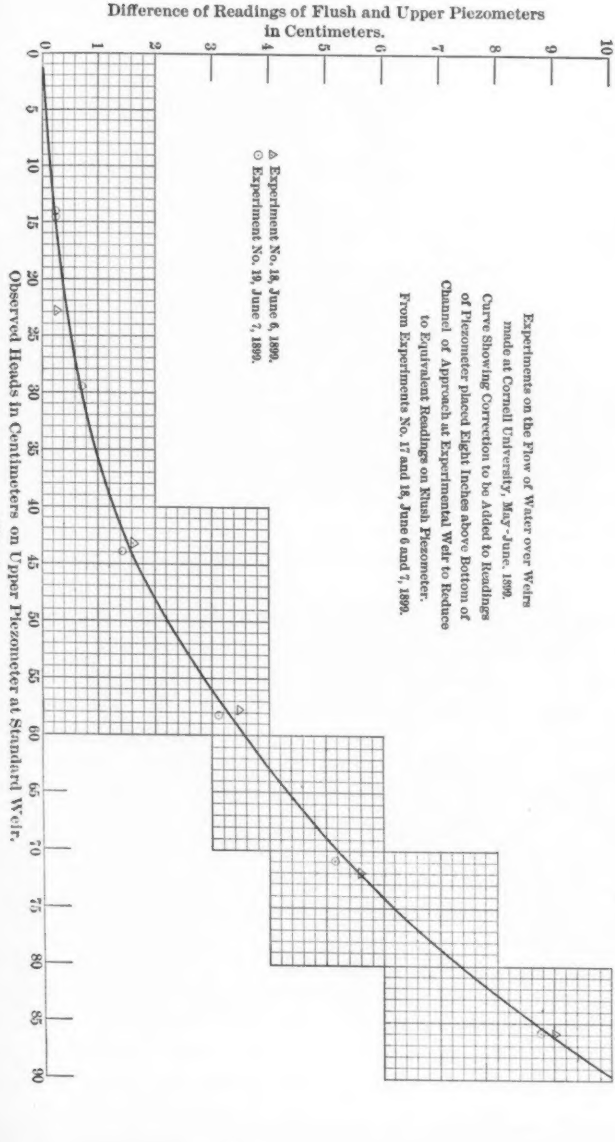
Difference of Readings of Flush and Upper Piezometers in Centimeters.
0 1 2 3 4 5 6 7 8 9 10
Experiments on the Flow of Water over Weirs made at Cornell University, May-June, 1899.
Curve Showing Correction to be Added to Readings of Piezometer placed Eight Inches above Bottom of Channel of Approach at Experimental Weir to Reduce to Equivalent Readings on Flush Piezometer.
From Experiments No. 17 and 18, June 6 and 7, 1899.
△ Experiment No. 18, June 6, 1899.
⊙ Experiment No. 19, June 7, 1899.
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90
Observed Heads in Centimeters on Upper Piezometer at Standard Weir.

Fig. 6.

The method used in reducing the experiments on weirs having crests of irregular profile is the same as for the sharp-crested weir, with the following exceptions:

In addition to the piezometric observations, direct observations of the head on each weir were taken on gauge boards situated above the standard and experimental weirs, respectively. The agreement between the heads so derived and those deduced from the mean piezometric readings is close in all cases, with the exception of the observed heads on the experimental weir in Experiments Nos. 1, 2, 3 and 4. Without going into the detail, it may be stated that for these four experiments, which were the first made, the heads directly observed on the gauge boards are apparently the more reliable, and they have accordingly been used in reducing these four experiments.

Again, in Experiments Nos. 1, 2, 3 and 4, no observations were taken on the middle piezometer at the standard weir. In order to reduce the observed heads as actually taken on the up-stream piezometer at this weir to equivalent heads on the middle piezometer, a correction has been applied, the value of which was obtained as follows: Plotting the difference between the readings of the upper and middle piezometers as ordinates for all experiments in which readings were taken on both, and using the observed heads on the up-stream piezometer as abscissas, a mean curve has been drawn, from which the correction to be applied to any reading on the up-stream piezometer can be read directly. This curve is shown in Fig. 5. The reason for using the readings of the middle, in preference to those taken on the up-stream piezometer at the standard weir, is that the former agree more closely, on the whole, with the readings of the gauge board; also, the middle piezometer, which is only 10 ft. distant from the bulkhead on which the standard 16-ft. weir was located, is more nearly at the proper distance from the weir. Moreover, the up-stream piezometer was situated so far back from the standard weir as to be evidently disturbed somewhat by the entrance velocity of the water in the leading channel.

With the exception of Experiments Nos. 18, 19, 20 and 21, the readings at the experimental weir were taken from a piezometer placed horizontally across the channel of approach at a height of about 8 ins. above the bottom. In order to reduce the readings from this piezometer to the equivalent readings from the piezometer placed

PLATE XIV.
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RAFTER ON FLOW OF WATER OVER DAMS.

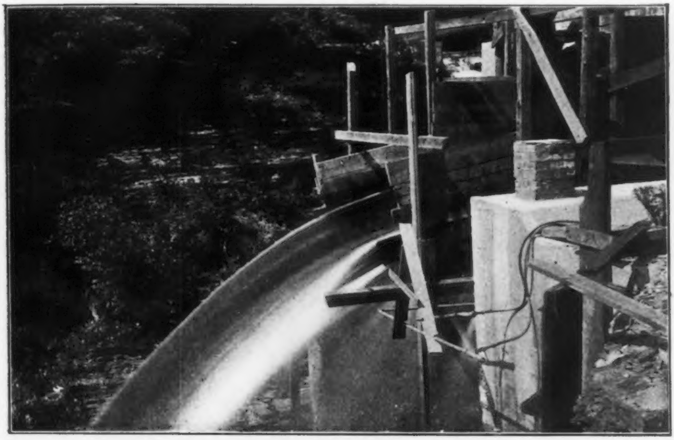


FIG. 1.—LOWER END OF CHANNEL. CORNELL UNIVERSITY HYDRAULIC LABORATORY.

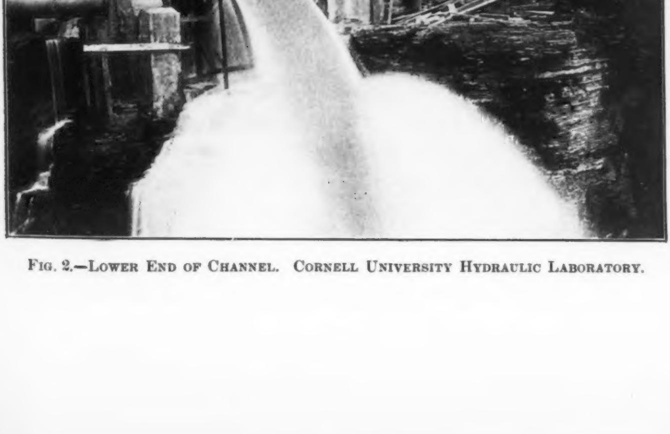


FIG. 2.—LOWER END OF CHANNEL. CORNELL UNIVERSITY HYDRAULIC LABORATORY.

flush with the bottom of the channel, a correction curve has been deduced in the following manner: In Experiments Nos. 18 and 19 observations were taken, both from the flush piezometer and from one situated 8 ins. above the bottom. Plotting the differences between the readings of these two piezometers corresponding to given heads on the standard weir as ordinates, and using the observed heads on the standard as abscissas, a mean curve has been drawn, from which a correction to be applied in any case can be read directly. This curve is shown in Fig. 6.

In regard to the use of this correction curve, it may be pointed out that the error resulting from the use of a piezometer placed otherwise than flush with the bottom or side of the channel is a function of the velocities in the channel of approach. Inasmuch as the discharging capacities of weirs of different sections vary greatly under the same heads, the velocity of approach at any given head will depend both

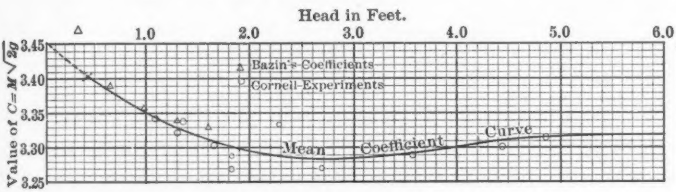


FIG. 7.

upon the height P of the weir and on the coefficient of discharge for the particular head and section of weir considered. Unfortunately, the data obtained were insufficient to enable the effect of these elements to be taken into consideration separately. Hence, the corrections obtained from the mean curve of Fig. 6 must be considered as approximate only. Some of the deviation of the experimental coefficients from the mean coefficient curves may undoubtedly be attributed to the uncertainty as to just the proper value of this correction.

Pages 267 to 284 show the coefficient curves finally fixed upon by the foregoing discussion for Cornell University Experiments Nos. 1 to 19, inclusive, and also the tabulations of the results. The coefficient curve for Experiments Nos. 20 and 21 on the standard sharp-crested weir is shown in Fig. 7. These curves are so self-explanatory as to render extended description unnecessary.

In the tabulations Column (1) shows the heads in feet, as read from the coefficient curves, Column (4) giving the discharge per lineal foot of crest in cubic feet per second. Columns (2) and (3) give the values of the coefficients M and C.

The photographs on Plate XIV show the lower end of the lower channel, as it appeared on the afternoon of June 3d, 1899, while experiments on the Rexford Flats section were in progress.

It is not the writer's intention to review extensively the results of the Cornell University Experiments at this time, any farther than to point out that they were, in the fullest sense, practical experiments.

In Experiments Nos. 7 and 8 an attempt was made to gain some idea of the effect of a rough surface on dams. In Experiment No. 7 the weir was of the usual form, with the crest constructed of planed matched pine, as already described, while in Experiment No. 8 the up-stream face of the crest was covered with ¼-in. mesh, galvanized wire screen. A comparison of these two experiments is very instructive. The upper limiting head of No. 7 was 4.996 ft. and of Experiment No. 8, 5.011 ft. For 5 ft. head, as determined from the curves, we have for Experiment No. 7, a discharge of 40.98 cu. ft. per second per lineal foot of crest, while for Experiment No. 8, 5 ft. head gives a discharge of 40.74 cu. ft. per second per lineal foot of crest. Similar comparisons at other heads show the effect of the wire screen to have been but slight. The result of this experiment was such as to lead to the conclusion that very little difference would be experienced in the flow over a dam after the first few months, during which time the planking, under the smoothing effect of the silt in flowing water, etc., may be expected to come substantially to the hydraulic condition of planed boards. Accordingly, as the time for completing the work was limited, no further determinations were made on this line.

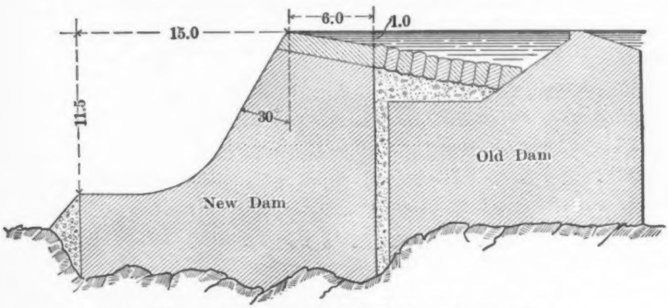
It is recognized, however, that but for the limitation of time under which the experiments were carried out, it would have been very desirable to have experimented somewhat farther on a number of additional forms of weirs, and it is to be hoped, in view of the vast practical importance of an accurate knowledge of flow over dams, that the Cornell University authorities will carry these experiments considerably farther, keeping especially well within the limits of actual practice in dam construction.

Application of Data to Cases in Practice.

We may now consider the application of the foregoing data to some of the dams at several of the gauging stations previously referred to.

Seneca River at Baldwinsville.—At Baldwinsville Station on Seneca River, there is a substantial masonry dam, as shown by Fig. 8. It was built in 1895, taking the place of an old crib dam located just above, as shown in the illustration. The crest is 423 ft. long, and is very nearly level. The catchment area of Seneca River, at Baldwinsville, is 3 103 sq. miles.

For most of the year flash boards, 1 ft. in height, are used on a portion of the crest. The flow over these has been computed by



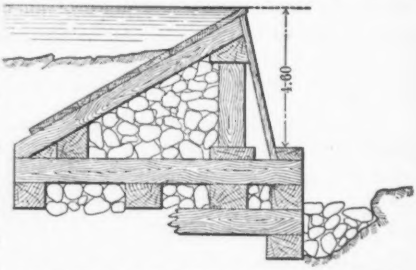
CROSS-SECTION OF DAM ON SENECA RIVER AT BALDWINSVILLE.

Fig. 8.

Francis' formula. The situation is somewhat complicated by the presence of the old dam. Taking into account the water cushion between the two crests, it was considered that Bazin's Series No. 115 would fairly apply, and, accordingly, the discharge curve was computed on this basis. The conditions here are so unusual that a special determination should be made as a check on the foregoing assumption. This was not done, during the Cornell University Experiments, for lack of time.

Oswego River at Fulton.—The catchment area above this dam is 4 916 sq. miles. There are extensive manufacturing establishments at the ends. The dam is a substantial masonry construction with a nearly vertical front, and with a back slope of 1 to 8. The

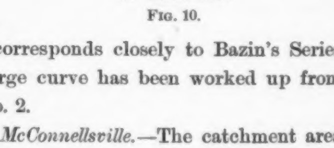
crest is slightly rounded. Bazin's section making the nearest approach to this is Series No. 117, which was recognized as being only an approximation. Cornell University Experiment No. 3 has furnished the data for working out a new discharge curve applying more nearly to the conditions of this dam than the original. It is believed that the revised curve gives the true discharge within a small percentage. At a depth of 2 ft. on the crest, the discharge, by the revised curve, is 9.8% less than by the original curve.



CROSS-SECTION OF DAM ON CHITTENANGO CREEK AT BRIDGEPORT.

FIG. 9.

Chittenango Creek at Bridgeport. — The cross-section of this dam is shown by Fig. 9. The catchment area above the point of gauging is 307 sq. miles, while the crest is 259.2 ft. in length. At the ends there are platforms over the bulkheads, and about 2.5 ft. above the main crest. The flow over these platforms when the water rises to their height, was originally computed by Bazin's Series No. 113, while Series No. 130 was applied to the main crest, shown by Fig. 9. The revised discharge curve for this dam is based upon Cornell University Experiments Nos. 1 and 10. The computed discharge, as per the revised curve, is—at a depth of 2 ft. on the crest—11.9% less than by the original curve.



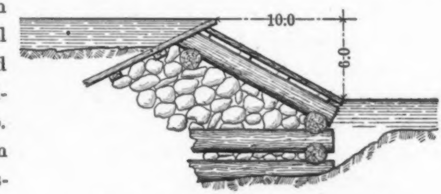
CROSS-SECTION OF DAM ON ONEIDA CREEK AT KENWOOD.

FIG. 10.

Oneida Creek at Kenwood.— The catchment area above this dam is 59 sq. miles. The crest is level, and is 79.4 ft. in length (see Fig. 10). The cross-section corresponds closely to Bazin's Series Nos. 130 or 135. The final discharge curve has been worked up from Cornell University Experiment No. 2.

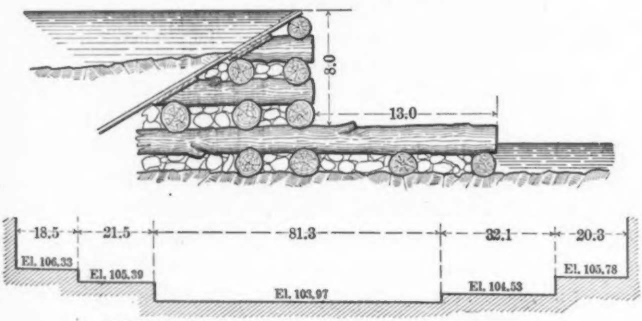
West Branch of Fish Creek at McConnellsville.—The catchment area above this dam is 187 sq. miles. The crest was originally quite irregular longitudinally, but was brought to a nearly uniform level by

spiking on strips of plank, which extended down the back or up-stream face. The length is 175.7 ft. (see Fig. 11). Bazin's Series No. 170 conforms in its general form closely to the cross-section, except that the projection of the planking of the back face over the front, forms an air space which has a disturbing effect on low flows. For minutely accurate results, on such a profile, special determinations should be made. Cornell University Experiment No. 7 has been used in preparing the final discharge curve.



CROSS-SECTION OF DAM ON WEST BRANCH OF FISH CREEK AT McCONNELLSVILLE.
FIG. 11.

East Branch of Fish Creek near Point Rock.—The catchment area above this dam is 104 sq. miles. The crest is 173.7 ft. in length, and is at several different heights, as shown in Fig. 12. Bazin's Series No. 130 applies closely. The final discharge curve is based upon Cornell University Experiment No. 1.

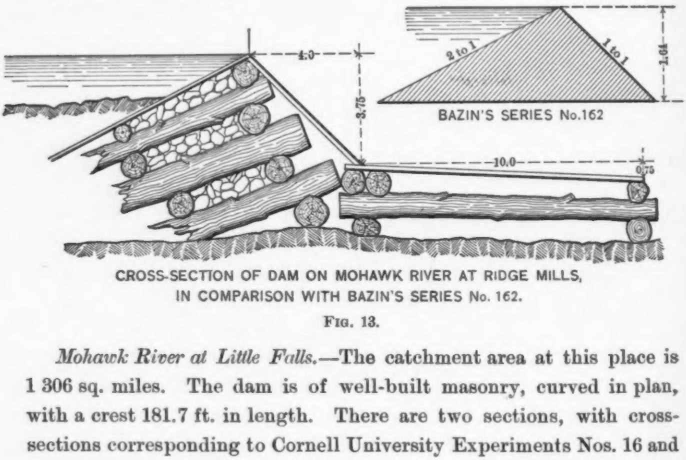


CROSS-SECTION AND PROFILE OF DAM ON EAST BRANCH OF FISH CREEK NEAR POINT ROCK.
FIG. 12.

Mohawk River at Ridge Mills.—The catchment area is 153 sq. miles. The crest is 122.7 ft. long, and is at three different elevations. As shown by Fig. 13, the experimental and actual sectional profiles agree closely.

Bazin's Series No. 162 is of such a form that the discharge is nearly

uniform at all heads. The syphon action of the sloping front face begins at low heads and continues to act with indefinitely increasing heads. There is no point where marked changes in regimen occur, as with depressed and adhering nappes. The coefficient of Cornell University Experiment No. 6 agrees closely with Bazin's No. 162. A discharge curve, based upon Bazin's No. 162, varies at 2 ft. depth on crest, only 1.5% from the discharge curve by Cornell University Experiment No. 6. Crests of this general form are especially applicable wherever accurate records of flow are required. For reasons given by Bazin, in the preceding abstracted matter, crests of this general form, but with flat upper surface, should be avoided. On this point see page 263 preceding.



CROSS-SECTION OF DAM ON MOHAWK RIVER AT RIDGE MILLS,
IN COMPARISON WITH BAZIN'S SERIES No. 162.

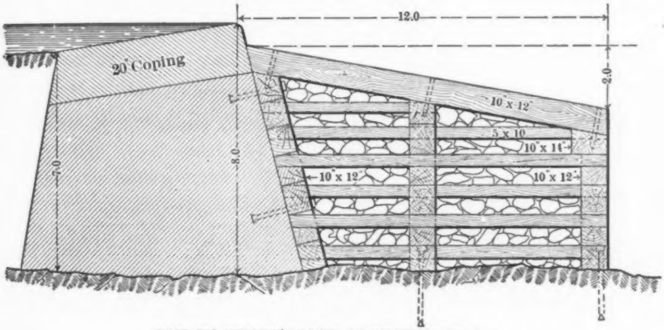
FIG. 13.

Mohawk River at Little Falls.—The catchment area at this place is 1 306 sq. miles. The dam is of well-built masonry, curved in plan, with a crest 181.7 ft. in length. There are two sections, with cross-sections corresponding to Cornell University Experiments Nos. 16 and 17, except that the dam has slopes on the front face for the two sections, 1 : 6 and 1 : 4, respectively. Inasmuch as there was full admission of air in the experiments, this fact would not affect the results, although the projection of the front face of the dam, itself, due to the slope may introduce disturbing elements in the flows, which would modify somewhat the results of Cornell University Experiments Nos. 16 and 17. The vertical front was used in the experiments in order to expedite the work.

The discharge curve worked out originally for this dam, was based upon Bazin's Series Nos. 117 and 135. A new curve, derived from

Cornell University Experiments Nos. 16 and 17, gives, for heads of 2 ft., 8.8% less flow than the original.

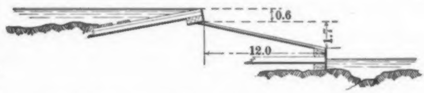
Mohawk River at Rexford Flats.—The catchment area at this place is 3 385 sq. miles. The dam is a substantial masonry construction with a timber apron, as shown by Fig. 14. The crest is 675 ft. long. The filling at the back of the dam makes, in effect, a long flat crest on the



DAM ON MOHAWK RIVER AT REXFORD FLATS.
FIG. 14.

up-stream side. In the original discharge curve, Bazin's Series Nos. 117 and 141, were taken as applying best. This dam was experimented upon at Cornell University, the new discharge curve resulting therefrom agreeing very closely with the original curve. The variation in the two curves at 2 ft. depth on the crest is only about 1 per cent.

Oriskany Creek at Oriskany.—The catchment area here is 144 sq. miles. The crest is 214 ft. in length, and is at three different elevations (see Fig. 15). For the original discharge curve, a mean of the coefficients of Bazin's Series Nos. 117 and 141 were considered to apply best. A revised discharge curve, based upon Cornell University Experiment No. 14, has been worked out. At a depth of 2 ft. the new curve increases the discharge 1.7 per cent.

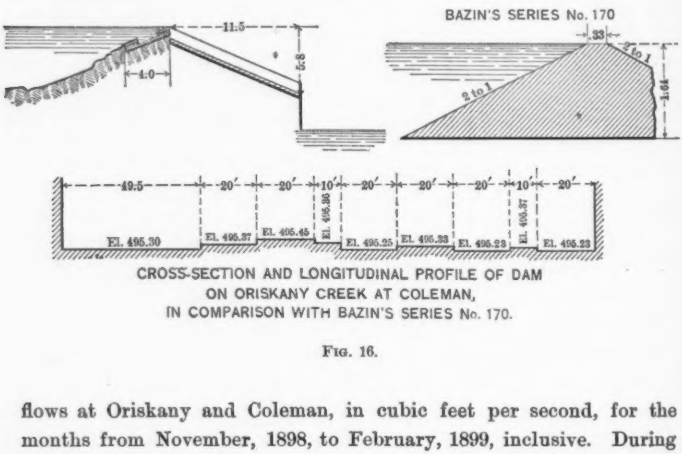


CROSS-SECTION OF DAM ON ORISKANY CREEK AT ORISKANY.
FIG. 15.

Oriskany Creek at Coleman.—This station is a little more than a mile above the dam at Oriskany, just described. The catchment area is

141 sq. miles, or 3 sq. miles (2.2%) less. Fig. 16 shows the cross-section in comparison with Bazin's Series No. 170, as well as the irregularities of the crest longitudinally. The remarks previously made as to flow, on Bazin's Series No. 162, apply to No. 170 and other sections of similar form. The disturbing effect of the departure from the theoretical form is unknown in this case, the same as for the dam on Mohawk River at Ridge Mills.

The object of establishing two stations on Oriskany Creek was to determine whether on dams of different forms, but with nearly the same catchment areas, the flows could be gauged closely enough to give fairly comparable figures. The following tabulation gives the



CROSS-SECTION AND LONGITUDINAL PROFILE OF DAM ON ORISKANY CREEK AT COLEMAN, IN COMPARISON WITH BAZIN'S SERIES No. 170.

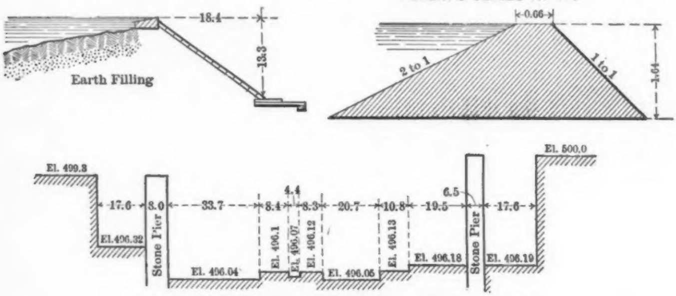
Fig. 16.

flows at Oriskany and Coleman, in cubic feet per second, for the months from November, 1898, to February, 1899, inclusive. During the frozen months of this period, the ice was kept clear for several feet back from the crest of each dam. The results show fair agreement, and indicate, that, even when one of the cases is complicated, as at Coleman, by discharge through several water wheels, comparable results may still be gained.

Month.	Discharge at Oriskany.	Discharge at Coleman.
November	327	306
December	327	335
January	295	297
February	291	283

Saquoit Creek at New York Mills.—The catchment area here is 52 sq. miles. The crest is as shown by Fig. 17. Bazin's Series No. 175 is taken as applying best to the main section. For the flash boards at the end sections, Francis' formula has been used.

West Canada Creek at Middleville.—The catchment area above this dam is 519 sq. miles. The crest is 330.5 ft. in length, and is leveled up, as described for the dam at McConnellsville (see Fig. 18). The original discharge curve was based upon Bazin's Series No. 170. A new curve based on Cornell University Experiment No. 15 (Rexford Flats section with rounded corner) gives 23% less discharge at 2 ft. depth than the original. The writer considers that this section should be specially determined, for accurate results, and the foregoing



CROSS-SECTION AND LONGITUDINAL PROFILE OF DAM ON SAQUOIT CREEK AT NEW YORK MILLS, IN COMPARISON WITH BAZIN'S SERIES No. 175.

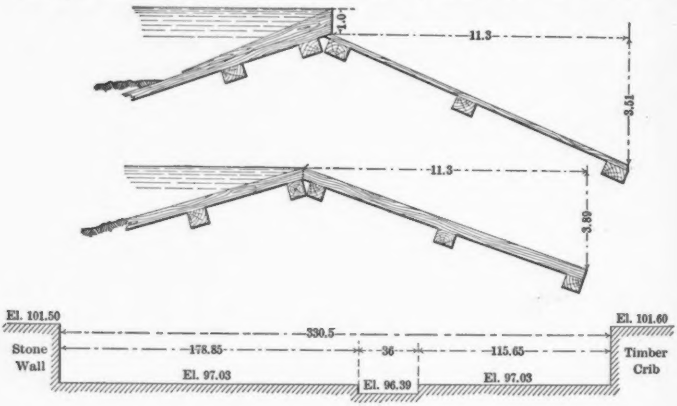
FIG. 17.

discrepancy is cited merely to show how useless it is, if accurate results are required, to apply the nearest form at hand. The whole study shows that, frequently, what appears at first sight to be relatively unimportant produces very marked changes in discharge.

Cayadutta Creek near Johnstown.—The tributary catchment area is 40 sq. miles. The main section, Fig. 19, is quite different from any of the dams thus far considered. Bazin's Series No. 130, was taken as being nearer than any other, while Series No. 115 was applied to the bulkheads at the ends. Where a high degree of accuracy is required for gaugings over a nondescript section of this sort, special experiments must be made.

Schoharie Creek at Fort Hunter.—The catchment area above this dam is 947 sq. miles. As shown by Fig. 20, it has a sectional profile similar to those of the dams on Oriskany and West Canada Creeks. The original discharge curve was based upon Bazin's Series Nos. 117 and 141. A re-computation, using Cornell University Experiment No. 14, gives substantially the same curve.

The foregoing account of several applications of the new views as to flow over dams has been made as concise as possible in order not to lengthen this paper unnecessarily. Matters of interest relating to the leakage of dams and flumes, methods of computing discharge through nondescript water wheels, and many other questions, are pur-



CROSS SECTIONS AND PROFILE OF DAM ON WEST CANADA CREEK AT MIDDLEVILLE.

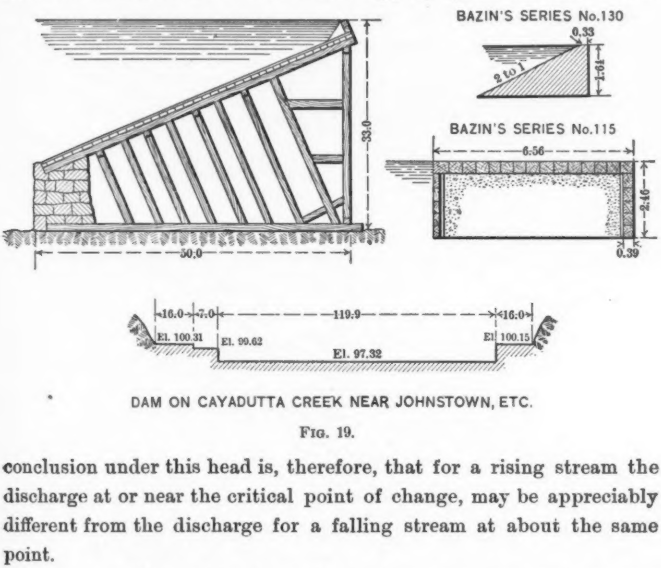
Fig. 18.

posely left untouched, in order to discuss more thoroughly the main question of how to compute the flow over dams. The Schoharie Creek dam may be especially mentioned as one with considerable leakage, and which is used here for illustrative purposes only.

In passing, it may be remarked that one result of the Cornell University Experiments was to show that the discharges, as per Bazin's Series Nos. 130 and 135, were much too high, especially at the considerable heads occurring at several of these gauging stations. The reasons for this are found, apparently, in the high discharges accompanying the depressed and adherent nappes, which occur at the low

heads Bazin experimented with. This, Bazin himself has pointed out, in the matter abstracted from his last paper, on a previous page. The great difference, however, is only fully realized when we carry out comparable experiments to the heads used at Cornell University.

The different methods of experimentation may also be taken into account. Bazin usually began with a low head, gradually increasing to the higher; whereas, at Cornell University, in all the experiments, the high heads were run first, and were gradually reduced. In both cases, the established regimen of flow, whatever it may have been, was continued longer than would have occurred under the contrary condition, the coefficients for the two states lapping by one another.* The



DAM ON CAYADUTTA CREEK NEAR JOHNSTOWN, ETC.

Fig. 19.

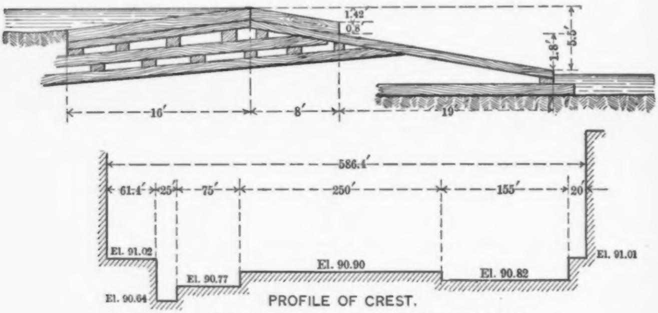
conclusion under this head is, therefore, that for a rising stream the discharge at or near the critical point of change, may be appreciably different from the discharge for a falling stream at about the same point.

By way of illustrating the method of computation used, we may discuss the computation for Schoharie Creek dam. Table No. 4 shows how the data for the discharge curve for this dam have been arrived at, the coefficients used therein being derived from Cornell University Experiment No. 14. To begin with, zero of the crest gauge is at elevation 90.68 ft. The crest itself divides into a series of sec-

* As to just the condition of the nappes for Bazin's Series Nos. 130 and 135, and several similar sections, see the tabulations of Bazin's Series, on preceding pages.

tions, with elevations as shown on the longitudinal profile of Fig. 20, and which are designated in the table, by the letters A, B, C, D, E, and F.

The method of procedure for the computation of points for the discharge curve is as follows: The average elevation of each section of dam—A, B, C, etc.—having been computed with reference to zero of the crest gauge, the depth of water flowing over each section, corresponding to a series of readings on the gauge, was deduced and tabulated, as shown. Thus, for section A, we have, in Column (4), head on section in feet, and so on for the other sections. Column (4) also includes the discharge per lineal foot of crest, for heads ranging from 0.2 ft. up to 8.0 ft., together with the total flow per section, for the same heads. These computations are made on the basis of



SECTION AND PROFILE OF SCHOHARIE CREEK DAM.

Fig. 20.

no end-contraction at the ends of the sections. The summation at the foot of the table represents the total flow over the dam, and ranges from 26 cu. ft. per second, for a head of 0.2 ft. to 41 182 cu. ft. per second, for a head of 8.0 ft. The discharge curve is constructed by plotting the final footings, with the heads on the crest in feet as ordinates, and discharges in cubic feet per second, as abscissas.

The foregoing general method has been applied, with necessary variations to fit each special case, to all the gauging stations herein referred to.

On examining the values of $C = m\sqrt{2g}$ in the coefficient tables given herewith, the great range in discharge, not only for different

TABLE No. 4.—SHOWING METHOD OF COMPUTING DISCHARGE CURVE FOR DAM ON SCHOHARIE CREEK AT FORT HUNTER.

Discharge Coefficients are from Cornell University Experiment No. 14.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Section.	Length, in feet.	Elevation, in feet.	Head above gauge zero.	0.2	0.6	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
A........	20	91.01	Head on Section *A*, feet.....	......	0.27	0.67	1.67	2.67	3.67	4.67	5.67	6.67	7.67
			Discharge per foot of crest.	......	0.37	1.62	6.82	14.13	22.73	32.62	43.62	55.80	68.30
			Total flow over section......	......	7.40	32.40	136.00	283.00	455.00	652.00	872.00	1 116.00	1 366.00
B........	155	90.82	Head on Section *B*, feet.....	0.06	0.46	0.86	1.86	2.86	3.86	4.86	5.86	6.86	7.86
			Discharge per foot of crest.	0.06	0.90	2.40	8.20	15.70	24.65	34.65	45.90	58.00	70.85
			Total flow over section......	9.30	139.50	372.00	1 271.00	2 434.00	3 821.00	5 371 00	7 115.00	8 990.00	10 982.06
C........	250	90.90	Head on Section *C*, feet.....	......	0.38	0.78	1.78	2.78	3.78	4.78	5.78	6.78	7.78
			Discharge per foot of crest.	......	0.70	2.15	7.60	15.10	23.90	33.95	45.00	57.05	69.80
			Total flow over section......	......	175.00	538.00	1 900.00	3 775.00	5 975.00	8 488.00	11 250.00	14 263.00	17 450.00
D........	75	90.77	Head on Section *D*, feet.....	0.11	0.51	0.91	1.91	2.91	3.91	4.91	5.91	6.91	7.91
			Discharge per foot of crest.	0.10	1.10	2.62	8.46	16.20	25.05	35.20	46.50	58.50	71.50
			Total flow over section......	7.50	83.00	196.00	634.00	1 215.00	1 879.00	2 640.00	3 488.00	4 388.00	5 363.00
E........	25	90.64	Head on Section *E*, feet.....	0.24	0.64	1.04	2.04	3.04	4.04	5.04	6.04	7.04	8.04
			Discharge per foot of crest.	0.36	1.77	3.28	9.40	17.20	26.40	36.60	48.05	60.30	73.30
			Total flow over section......	9.00	44.00	82.00	235.00	430.00	660.00	915.00	1 201.00	1 508.00	1 833.00
F........	61.4	91.02	Head on Section *F*, feet.....	......	0.26	0.66	1.66	2.66	3.66	4.66	5.66	6.66	7.66
			Discharge per foot of crest.	......	0.36	1.60	6.80	14.10	22.70	32.58	43.58	55.75	68.20
			Total flow over section......	......	22.00	98.00	417.00	865.00	1 394.00	2 000.00	2 676.00	3 423.00	4 187.00
Total flow over dam, in cubic feet per second............				26.00	471.00	1 318.00	4 593.00	9 002.00	14 184.00	20 066.00	26 602.00	33 688.00	41 182.00

Elevation of zero of crest gauge = 90.68 ft.

forms of dams, but for varying heads, becomes apparent. These tables indicate that the making of accurate gaugings over dams demands considerable skill in the application of the available information. For minutely accurate results, special experimentation is, in many cases, indispensable.

As regards experiments on dams, the Cornell University Hydraulic Laboratory can hardly be improved upon, and the University authorities deserve the sincere thanks of every hydraulician, for furnishing an equipment of this character. It is to be hoped that data of flow over dams may be greatly extended there in the next few years.

In concluding the paper, the writer may remark that the studies herein discussed were, in reality, only a side issue of the entire investigation of the water-supply problems carried out for the United States Board of Engineers on Deep Waterways. For whatever deficiencies may appear, the writer hopes he may be pardoned, because of the time limit set by the Board, which was, the completion of everything within one year. This condition compelled a strictly business-like administration and the omission of much purely scientific detail which, with more time available, it would have been very pleasant to pursue somewhat farther. It was necessary, indeed, under the requirements of the Board, to be first of all a business man—driving the matter in hand along rapidly to a final conclusion—and only indulging in pure scientific work so far as this did not conflict with definite progress from day to day.

As regards the Cornell University authorities, the conditions are different, and they will without doubt ultimately supply the engineering profession with far more extended knowledge of the flow of water over dams, especially at high heads, than is now possessed.

The total cost of this set of experiments, including materials, common labor, carpenters, engineering assistants, draughtsmen, stenographer and time of writer did not exceed $1 800. This figure does not include either Professor Williams' time or cost of gauges, which were paid for by the University as part of the permanent equipment, but it includes all payments on account of these experiments by the United States Board of Engineers on Deep Waterways.

The writer's thanks are due to Professor E. A. Fuertes, Director of Cornell University College of Civil Engineering, for many courtesies received during the progress of the study.

DISCUSSION.

GEORGE Y. WISNER, M. Am. Soc. C. E. (by letter).—In the investigation for developing a project for a deep waterway from the Great Lakes to the Atlantic, which was commenced by the United States Board of Engineers on Deep Waterways in the fall of 1897, it became evident, from the start, that the existing data relative to the flow of water over dams were inadequate for the accurate determination of river discharge where the depth on the crest of the dam was much over 1.5 ft. Mr. Wisner.

The uncertainty as to the value of the coefficients of weir formulas which should be used for dams of different cross-sections, and for different depths on the crest, made it apparent that additional investigations would be necessary before satisfactory estimates could be made of the value of water-power rights which may be modified, or of the amount of slope walls and bank protection which would be needed between the limits of the high and low-water stages of the proposed waterway.

At first, it was thought that extended observations with modern current meters at several of the principal dams in question would be necessary, but since the coefficient of the weir formula varies greatly with the shape of the dam, and as there are but few of the dams on the Oswego, Mohawk and Hudson Rivers which have similar cross-sections, such a method would have been incomplete and unsatisfactory, and was not attempted.

In the fall of 1898 the experiments of H. Bazin, published in the *Annales des Ponts et Chaussées*, became available, and established the coefficients for a great variety of different-shaped weirs, but, unfortunately, for depths of less than 1.5 ft. on the crests.

Soon after the appearance of Bazin's final paper, the writer entered into correspondence with Professor Williams, and later with Mr. Rafter, with a view to utilizing the Hydraulic Laboratory at Cornell University for the extension of the Bazin experiments to greater heads. The work was finally ordered in the following April, Messrs. Rafter and Williams acting in conjunction. Mr. Williams' special work was the installation of the necessary measuring apparatus and the measurement of the heads on the weirs during the experiments.

It was not expected that a high degree of precision would be attained, but it was believed the results would suffice for determining with all needful accuracy the floods of the Mohawk and Hudson, from the observed heights on the several dams, and also for the preliminary design of the Lake Erie Regulating Works. The experiments have answered these requirements, and are exact enough for application to any similar practical case.

In authorizing these experiments, the Board of Engineers on Deep

Mr. Wisner. Waterways insisted that the standard sharp-crested weir should be thoroughly calibrated for all depths on the crest for which used. This, however, the observers were unable to accomplish in a satisfactory manner, but it is to be hoped this will be done in time to embody the results in this discussion.

Referring to the mean coefficient curve of the standard weir shown by Fig. 7, it will be noted that for heads of from 2 to 3 ft., the coefficient is a minimum. So far as the writer is aware, there is no good reason for this, and it is probable that the peculiar shape of the curve may be due to incorrect calibration of the standard weir, and to the effect of velocity of approach and to side walls of a narrow flume for which the correction made is, apparently, largely a matter of judgment.

An examination of the curves on pages 267 to 284, inclusive, shows the same peculiarity for a large percentage of the experiments, and a study of the data and results indicates that incorrect determination of the effect of velocity of approach at the experimental weir and unknown resistances of the side walls of a narrow flume are the principal causes. Comparing the curve on page 284 with that in Fig. 7, there is apparently no reason why one should be a regular curve and the other not, and, as observations made subsequent to those discussed in the paper indicate that the former is correct, it is a fair presumption that additional observations will very likely modify the shape of the latter.

Mr. Williams. GARDNER S. WILLIAMS, M. Am. Soc. C. E.—It would afford the speaker great pleasure were he able to give nothing but commendation for the work of the author. Unfortunately, he is compelled to differ from the opinions expressed in the paper, on some rather essential points. He is fully aware that any criticisms that may be made, so far as they relate to the execution of the experiments, will be criticisms upon himself, and, in extenuation, has only to say that the institution which he has the honor to represent is searching for the truth, and will be glad, as occasion comes, to point out its own errors as well as those of others. It will welcome any criticisms, any suggestions, any instructions.

At the outset of the investigation described, it was decided that as the work was intended to be an extension of that of Bazin, the conditions of his experiments would be conformed to as nearly as possible, and his formula be used in the reductions. For this reason the experimental weirs were made 2 m. long, and the piezometers for reading the heads were located at distances from the experimental weirs bearing approximately the same ratio to their height that the distance to the location of Bazin's point of reading head bore to the height of his weirs. Bazin's formula* differs from the formulas of Francis, Fteley and Stearns, and Hamilton Smith, in that it provides

* *Annales des Ponts et Chaussées*, October, 1888.
Proceedings, Engineers' Club, Philadelphia, 1890, Vol. vii, p. 251 *et seq.*

Mr. Williams.

for the effect of velocity of approach in its coefficient; whereas all the others require that the observed head shall be corrected for the velocity of approach, so that, while the Francis coefficient, 3.33, is to be applied to the head after it is thus corrected, the Bazin coefficient is to be applied to the head as it is observed.

The statement of the author that by reference to Table No. 2 it will be seen that there is very little change in the coefficient after a height of weir of 6½ ft. is reached, also needs some decided modification, because the coefficient changes as the velocity of approach changes, and while the statement is true with the comparatively low heads, up to 1.6 ft., included in the table, it is not true with higher heads, as, for example, with a head of 100 cm., that is, about 3¼ ft., on a weir 11 ft. high, the discharge is about 0.6 of 1% greater than it is on a weir 13 ft. high, according to Bazin's formula.

Of course, it is fully realized that in speaking of a matter of 0.6 of 1% in a weir measurement it is getting down rather fine; but we are to consider that accuracy in the measurement of water for power purposes or for consumption and accuracy in the design of a crest to discharge the flood volumes of a stream, are two different things. In the latter case, if one comes within 5%, he is doing very well. In the former, one should get down as near to 1% as possible.

The author, apparently, makes a misstatement on page 292,* where he says that n—which was originally designated by the Greek letter μ by Bazin—is a coefficient which depends upon the height of the weir; now that is less than half the truth. It depends upon the height of the weir and the head over the weir,† and it is given quite accurately by the formula $\mu = 0.405 + \frac{0.00984}{h}$ for English units. That is, it is given satisfactorily by a formula which does not involve the height of the weir at all. In other words, in speaking generally, the coefficient μ varies with the head over the weir and not with its height. The coefficient m varies with the head over the weir and the height of the weir, and m is the factor which enters finally into Bazin's formula, the formula being virtually made up of three formulas, first a formula for μ, then one for m, which involves μ, and then one for Q which involves m.

In the reduction of Experiments Nos. 20 and 21 on pages 293 and 294, if the speaker understands the author correctly, a curve has been plotted for the discharge of the upper weir by Bazin's formula, as given on page 292, which contains the coefficient providing for velocity of approach. From this the author has taken a value for the discharge. He has then from that discharge computed the velocity of approach, added to the observed head the correction for the head due to this velocity of approach, taken the value of Q for this increased head

*Since this discussion was received, the statement in the original paper has been changed and now reads:

n = a coefficient which depends on p and h (p = height of crest of weir above bottom of channel of approach; and h = observed head on crest).

† *Annales des Ponts et Chaussées*, Oct., 1888, p. 445, and *Proceedings*, Engineers' Club of Philadelphia, 1890, p. 308.

Mr. Williams. from the Bazin curve again, and he says that generally two applications of the process were sufficient. In the speaker's judgment, inasmuch as each application was adding a velocity head that did not belong there, it seems that two applications should have been sufficient. If the author desired to determine the discharge for a weir under this head with no velocity of approach, the correction should have been subtracted, not added, and if he wished to obtain the head required on a weir with no velocity of approach to deliver the same quantity of water the correction for velocity of approach should have been made but once.

It is possible that the speaker has misunderstood the author's method of reduction, but he has taken pains to refer these statements to several others, conversant with Bazin's formula and with hydraulics in general, and it has been generally agreed that the language is misleading, if the process has not actually been so. That is, the language simply means that instead of using the observed head h which should have been used with Bazin's formula, the head $h + h_a$, as in the Francis formula, was used, the result being that the computed discharge of the standard weir by Bazin's formula is thereby made too large. It may be pertinent to enquire why, if the author deemed a correction for velocity of approach necessary at the upper weir, he did not also apply one at the lower weir where the velocity was several times as great. The author does not appear to have understood clearly the import of the discussion by Messrs. Fteley and Stearns upon the position for reading heads upon a weir, judging from the statement on page 294 that: "Messrs. Fteley and Stearns have pointed out that for standard sharp-crested weirs the head should be measured about 6 ft. back from the crest," for, by turning to the paper by Messrs. Fteley and Stearns,* the following statement will be found :

"The head, if measured outside of the angle of pressure, should be taken far enough up stream from the weir to represent the height of the water surface above the beginning of the surface curvature, *i. e.*, at a distance from the weir equal to 2½ times its height above the bottom of the channel."

Our weir was 13 ft. high, and the head, according to this, should have been read about 32 ft. up stream.

This brings us quite properly to the subject of weir experiments in general, and in the discussion of any hydraulic problem it is well to go back to the beginning and find out how much we really know about the thing in hand. We are dealing with weirs with end contractions suppressed, and so far as experiments have gone upon such weirs of a sufficient size to be compared with those which are discussed in this paper, in which the discharge has been measured volumetrically, the entire series of experiments is embraced in three investigations shown in Table No. 5.

* *Transactions*, Am. Soc. C. E., Vol. xii, p. 47.

TABLE No. 5.—EXPERIMENTS UPON WEIRS WITH END CONTRACTIONS SUPPRESSED, IN WHICH THE DISCHARGE WAS MEASURED VOLUMETRICALLY. Mr. Williams.

Observer.	Number of experiments.	Length of weir, in feet.	Height of weir, in feet.	Range of head, in feet.
James B. Francis....	17	9.995	4.60	0.73620 to 1.0600
Fteley and Stearns...	30	4.999	3.56	0.0746 " 0.8193
" " ...	10	18.996	6.55	0.4685 " 1.6038
Henry Bazin..........	67	6.562	3.72	0.194 " 1.012
" "	38	3.281	3.72	0.188 " 1.338
" "	48	1.640	3.296	0.191 " 1.779

It will be seen that Bazin's first series included nearly as many experiments as those of all the other investigators, and that, altogether, he has given us three times as many determinations of the flow over suppressed weirs volumetrically as the others have.

There is an important distinction between the methods of measuring head in Bazin's experiments and in the experiments of the American investigators. The latter adopted a position for reading the head 6 ft. up stream from the crest of the weir and about its level. Bazin read it 16.3 ft. up stream and at the bottom of the channel. The American experimenters took the water through a small opening in the side, in no case more than ⅜ sq. in. in area, which communicated with a pail in which the surface was read by a hook gauge. Bazin used an opening 4 ins. in diameter which communicated to a chamber built alongside of his canal in which the head was read by a hook gauge. Now, it will be realized at once that it is to be expected that the velocity of the water flowing toward the weir would be greater at the American position than it would be 10 ft. further up stream, and, as any increase of velocity head or of velocity means a corresponding decrease of pressure head, it may be expected that for the same observation, if the head were measured at the American position, it would appear to be lower for the same discharge than if it were measured at Bazin's position. Therefore, for a given head, we should expect that Bazin would show a less discharge than would the American investigators.

Some may be inclined to doubt the importance of the variation in position in reading heads. Upon that point it may be said that in some investigations carried on last summer, the head was read directly at the crest of the weir by means of a tube set in the weir itself and communicating with the crest by small openings 6 ins. apart. These openings were about ¼ in. in diameter, and were bored vertically at the exact crest of the weir, which was the section adopted by the United States Board of Engineers on Deep Waterways for the proposed regulating weir on Lake Erie, *i. e.*, No. 19 of the author's series. At the

Mr. Williams. time that these heads were read a tape was tacked upon the wall of the canal vertically at the crest, so that the top of the sheet could be read thereon at the same time that the pressure in the piezometer along the crest of the weir was read. With a head up stream of 97 cm. the tape read 70 cm. and the piezometer at the crest read 38 cm. There is the effect of velocity upon the head. The piezometer set on the crest of the weir read hardly more than half the depth of water which was actually flowing over the weir at that time, and as there were, altogether, somewhere in the neighborhood of thirty or forty experiments involving the crest piezometer, it may be affirmed that this was not an erratic observation.

TABLE No. 6.—Comparison of Observations and Formulas of Suppressed Weirs.

No.	Observer.	Weir.		Observation.		Discharge by Formula.			
		Length. Ft.	Height. Ft.	Head. Ft.	Discharge. Cubic feet per second.	Francis. Cubic feet per second.	Fteley & Stearns. Cubic feet per second.	Smith. Cubic feet per second.	Bazin. Cubic feet per second.
1...	J. B. Francis.........	9.995	4.60	0.9760	32.436	32.290	32.300	32.406	32.784
2...	Fteley & Stearns.....	18.996	6.55	1.4546	112.066	111.890	112.054	112.240	112.550
3...	" "	18.996	6.55	0.4685	20.178	20.306	20.175	....	20.890
4...	" "	4.999	3.56	0.8118	12.466	12.307	12.459	12.497	12.557
5...	" "	4.999	3.56	0.4569	5.199	5.162	5.212	....	5.326
6...	H. Bazin..............	6.562	3.72	0.9794	21.930	21.418	21.478	21.715	21.928
7...	"	6.562	3.72	0.5644	9.533	9.306	....	....	9.533
8...	"	1.640	3.296	1.0158	5.754	5.664	5.689	5.961	5.753
9...	"	1.640	3.296	0.5332	2.2005	2.1346	....	....	2.2002

2 Excess by Bazin's formula over Fteley & Stearns' measurement for $h = 1.4546$, is 0.43%
1 " " " " " Francis " " $h = 0.9760$, " 1.07%
4 " " " " " Fteley & Stearns " " $h = 0.8118$, " 0.71%
3 " " " " " " " " " $h = 0.4685$, " 3.52%
5 " " " " " " " " " $h = 0.4569$, " 2.44%
8 " " " measurement over Francis' formula " $h = 1.0158$, " 1.59%
6 " " " " " " " " $h = 0.9794$, " 2.39%
7 " " " " " " " " $h = 0.5644$, " 2.44%
9 " " " " " " " " $h = 0.5332$, " 3.09%

It may be said further, that, since this condition exists, it is possible to use such a form of weir as a Venturi meter, particularly when submerged, and there is no doubt that a series of coefficients for a weir of the form of the United States Deep Waterways Section might be given as such a meter that would compare quite favorably, in accuracy, at least with the coefficients given by the author for the various irregular weirs. Time does not now suffice to go into this to its fullest extent. It may be said, however, that as the crest is submerged the difference between the reading of the piezometer and the tape decreases, but they do not become equal up to 3-ft. heads, nor does the reading of the piezometer at the crest become equal to the depth of submergence within

submergences of 4 ft. Now, as stated before, Bazin's formula should be expected to give the higher head for a given discharge or a lower discharge at a given head than those of the American investigators; but when Bazin's formula is applied to the experiments of the American investigators, in which the discharge was measured, it universally gives a higher discharge than was observed. When the American formulas are applied to the American experiments they fit excellently; but when they are applied to Bazin's experiments they give a lower discharge than was observed. When we apply Bazin's formula to his own experiments, it fits most excellently, as is shown in Table No. 6. Mr. Williams.

Now, what is the meaning of this? Either Bazin had a different kind of water from that which the American experimenters had, or one or the other has done the better work. The question comes home at once: Which? Bearing upon this point, a criterion has been applied, the best that has occurred to the speaker thus far, to determine whether the experiments of the several investigators were homogeneous in themselves; that is, whether they would coincide or whether they would show erratic variations from one side to the other of some mean. The criterion was to take the measured Q's and from them to derive an $n^{\frac{2}{3}} h$, which, when plotted as an abscissa with the observed head as an ordinate, would give a straight line if n were constant. Of course, since n is not constant, but increases with the head at the higher heads, it does not give a straight line, but the variation is not great for the range of the experiments. That criterion showed clearly that the results of the experiments were homogeneous in themselves, and indicated a high degree of relative accuracy. That is, if one was right the other was right in the same series. Pains were then taken to study particularly the arrangement by which the quantity was measured, and the conclusion has been that the devices used by Francis and by Fteley and Stearns for starting and stopping the flow and also for determining the height of water in the measuring basin were more delicate, and capable of more accurate work than were those of Bazin, so that, patriotism aside, it seems that greater confidence may be reposed in the observations of Francis and of Fteley and Stearns than in those of Bazin, although the latter has made three times as many as the others. As already stated, the Francis formula gives results below Bazin for the lower heads, but the discharge curves of the two formulas cross at a head of about 1.4 ft. on a weir 11 ft. high, and above that Bazin gives lower discharges. Comparing the Francis formula with the discharges observed by Fteley and Stearns at the higher heads there is some evidence that the Francis formula gives too low results with such conditions, and therefore it seems that Bazin's formula is probably on this account the less accurate at the high heads.

Whether or not all wish to agree with the deductions as to the effect of velocity of approach or velocity past the openings and as to

Mr. Williams. the relative reliability of the work of the various investigators, they will probably agree that if Bazin's formula is to be used, the head should be measured as Bazin measured it, and if the Francis formula is to be used, the head should be measured as Francis measured it. But the question naturally arises, what difference does it make? A few days ago the speaker had the privilege of performing some experiments to see what difference it made whether the head was measured one way or another. The weir in question was a small decimal over 20 ft. in length. Its height was 5.85 ft. above the channel of approach. End contractions were suppressed. At a point 10.3 ft. up stream from the weir there was a pipe, 1 in. in internal diameter, set 1 ft. above the bottom of the channel of approach, transversely to the direction of flow. This pipe was perforated on its bottom with holes about $\frac{3}{16}$ in. in diameter every 3 ins. in its length. One end of this pipe was connected by means of a $\frac{3}{4}$-in. pipe and a $\frac{3}{4}$-in. hose, to a hook-gauge pail, which was set in a recess in the wall at a point about 6 ft. up stream from the weir. At the other side of the channel of approach was a similar recess. This transverse pipe was a device which was ordinarily used for measuring the head upon this weir, which, it may be said, incidentally, is a somewhat important one. As a result of the investigations at Cornell, the reliability of a measurement taken in that way was questioned; and, in order that there might be no mistake in the important work which was in hand, the plate which formed the side of the recess on the opposite side of the canal was tapped through at a point 6 ft. from the crest of the weir and 0.35 ft. below it, and a $\frac{3}{4}$-in. iron pipe screwed in, the face of which was filed off flush with the side of the channel. A wooden plug was then driven into the pipe and smoothed off flush; in this a $\frac{1}{4}$-in. hole, perpendicular to the side of the channel, was bored, thereby nearly reproducing the device used by Francis and by Fteley and Stearns. The transverse pipe was connected through the side of the canal to a second hook-gauge pail in the same chamber. The hook-gauge pail was removed, and in its stead was connected one of two glass tubes of $\frac{3}{8}$-in. inside diameter, which were mounted rigidly on a board in front of a common scale, divided in 2 mm. divisions. The other tube was connected to the new, or what will be designated as the Francis, piezometer. Two portable hook gauges were then clamped to the crest of the weir, and the water was raised to within about $\frac{1}{4}$ in. of the crest, and by measuring from the crest with the portable hooks the reading of the scale and of the permanent hook gauge for the crest of the weir, were determined by water level, so that there might be no mistake as to the setting of the instruments.

A series of investigations was then made to determine whether the tube which was connected to the transverse piezometer read in correspondence with the hook gauge, and it was found to do so with remarkable constancy for a wide range of head, the difference being

that the tube showed continuously about 0.003 ft. more head than the hook. It was therefore considered that any difference which appeared in the readings of the two glass columns would show the difference in head by the two piezometers. Omitting further detail, it was found that at low heads the transverse piezometer read high, and that at high heads it read low, and with a head of 2 ft. over the weir it made $2\frac{1}{2}\%$ difference in the discharge whether the reading was taken by the Francis piezometer or by transverse piezometer, the discharge by the Francis piezometer being $2\frac{1}{2}\%$ greater than by the transverse piezometer. That will give an idea of the importance, when using any formula, of measuring the head as it was measured when the formula was devised. If the head is measured some other way the formulas may or may not apply. Unfortunately, at Cornell, no provision was made in the construction of the plant for measuring the head upon the standard weir by either of the recognized methods. It was, therefore, necessary to resort to some other means, and, without knowing positively what the result would be, the transverse piezometer, which has been described by the author, and which coincides quite closely with that at the weir, which has just been discussed, was adopted. There was some suspicion when it was put in that it might lead to trouble, and it was expected that such checks could be made on the work as it went along as to detect such an error at once if it should occur. But the great pressure which was brought to bear to hurry the experiments, and the other duties which were demanded of the investigators, prevented the working up of those experiments, even the first of them, until the series was nearly completed. Then it became apparent that there was something wrong with the piezometers, and, accordingly, there was set, alongside the one at the lower weir, another, which was flush with the bottom of the channel of approach, and which probably coincided quite closely with Bazin's opening in the side of the canal at the bottom, although, of course, it did not coincide exactly. It was then found that, at the highest heads used, the difference in head, as measured by the two piezometers, amounted to about 10 cm., or about 0.3 ft., which means considerable in the discharge of the weir. The author has given, in Fig. 6, a correction curve which he applies in these experiments, and which is probably the best that could be done under the circumstances. The discovery of such an error at the lower weir led at once to the conclusion that there must be something the matter with the piezometer at the upper weir also, but time did not permit an investigation previous to the completion of the series of experiments described. As the speaker was, at the close of this investigation, requested by the Board of Engineers on Deep Waterways to continue the work by an investigation of the discharge over the "Deep Waterways" Section, so-called, Section 19, a rounded crest with a 45° up-stream slope, at various degrees of submergence, and also

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Mr. Williams. with free discharge, it gave an opportunity to make some further investigations upon the foregoing question, and in order to measure the head in another way at the standard weir three pipes were set longitudinally, that is, parallel to the direction of the flow of the water at a point 6 ft. above the bottom of the canal and 28 ft. up stream from the weir and with their down-stream ends projecting about 6 ins. down stream from the plane of the old up-stream transverse piezometer. These pipes were 6 ft. long, $\frac{3}{4}$ in. in diameter, nominally, and were perforated for 1 ft. at their down-stream end with holes on the quarter, so that there were around the pipe rings of four holes, each 1 in. apart for a distance of 1 ft. at the lower end. These pipes were connected together at the down-stream end, the up-stream end being plugged. A $\frac{3}{4}$-in. pipe connection, similar to those of the transverse piezometers, was carried through the bulkhead, so that they might be connected to one side of the tube gauge. The reason for adopting this type of piezometer was that Mr. FitzGerald* records that he investigated the head, when measured in pipes, by a device of this sort in which the perforated pipe is laid upon the bottom of the large pipe, in comparison with the head as given by a piezometer consisting of a chamber surrounding the pipe and communicating with it by holes in a plane at right angles to the axis of the pipe, and he found that there was apparently no difference. So that on the strength of those experiments it was ventured to assume that this would probably give a correct reading of head at the point where it was wished to measure it, *i. e.*, at a point corresponding to Bazin's position, if there is such a thing as a correct reading of head. About 40 experiments were made and they showed, at the highest heads observed, a little over 3 ft., that there was a difference of 3 cm. in the head as read by the transverse piezometer and as read by the longitudinal piezometer, the new longitudinal piezometer giving a head 3 cm. higher than did the old up-stream transverse piezometer. As soon as these data were obtained a memorandum of it, sufficient to locate a correction curve, was furnished to the author, who, however, decided to reject the readings of the up-stream piezometer and to adopt those of the middle one, which was 10 ft. back from the weir. Now, having simultaneous observations on the middle piezometer and the up-stream piezometer, and having a series comparing the new piezometer with the up-stream piezometer taken through the later investigation, the new piezometer readings, assumed to be correct, were plotted as a straight line at an angle of 45° with the axes on which the heads were laid off, and with this the old transverse up-stream piezometer and the middle piezometer were plotted. The differences could not be detected at a head of 5 cm., $\frac{1}{6}$ ft., but at 3.3 ft. the difference between the new and the old upper piezometer was 3 cm., the latter being low at all

* "Flow of Water in a 48-in. Pipe," *Transactions*, Am. Soc. C. E., Vol. xxxv, p. 259.

TABLE No. 7.—COMPARISON OF SIMULTANEOUS DISCHARGE OVER UPPER AND LOWER STANDARD WEIRS BY BAZIN'S FORMULA. Mr. Williams.

Experiment No.	UPPER WEIR. 16 ft. long, 13.13 ft. high.			LOWER WEIR. 6.56 ft. long, 5.2 ft. high.		Percentage of excess of lower over upper weir.
	Obs. h. Transverse Piezometer. 27 ft. up stream.	Cor. h Longitudinal Piezometer. 27 ft. up stream.	Q = cubic meters per second.	Obs. h. Flush Piezometer. 37 ft. up stream.	Q = cubic meters per second.	
	Cm.	Cm.		Cm.		
13	12.075	12.28	0.3998	22.744	0.4053	+1.378
12	15.023	15.30	0.5499	27.855	0.5484	−0.278
6	18.069	18.39	0.7178	33.175	0.7124	−0.203
11	21.294	21.65	0.9133	39.419	0.9214	+0.886
7	23.759	24.16	1.0720	44.000	1.0915	+1.820
10	26.810	27.21	1.2785	49.699	1.3305	+4.067
1	29.658	30.16	1.4900	55.213	1.5456	+3.732
8	29.723	30.22	1.4945	55.128	1.5436	+3.283
9	37.437	37.90	2.0805	68.238	2.1440	+3.052
5	43.812	44.22	2.6280	80.566	2.7772	+5.677
4	58.110	59.00	4.4800	105.639	4.2645	−4.810
3	72.710	74.22	5.7300	130.286	5.9582	+3.981
2	79.565	81.69	6.6200	142.557	6.8881	+4.041

points. The curve of the middle piezometer started above the longitudinal one and reached a maximum difference at a head of about $2\frac{1}{2}$ ft., where it was about 1 cm. high, and then dropped rapidly, appearing to cross the longitudinal piezometer at about 3.3 ft., after which it would read low. Now, of course, it cannot be affirmed that the new piezometer is a correct one to use with Bazin's formula, but, in view of the experiments of Mr. FitzGerald on the Rosemary Syphon, and in view of the comparison by the tube gauge of the transverse piezometer and the Francis piezometer it seems safe to infer that the new piezometer is the best one to use for the heads on this weir, or is as nearly correct as anything that we have. Furthermore, it was discovered that the head, as given by the new longitudinal piezometer, was much more steady and there was less vibration, less change and less pulsation than in the head as observed by the other. In order to complete the whole subject, an attempt was made to determine whether the manner of opening the gates, or the way in which the water was admitted to the chamber, would have any effect on the correction, and to that end all the head gates were opened a short distance, allowing the water to enter at the bottom of the chamber and flow over the weir. Then, at times, only two gates were opened, and they were opened wide, so that the water would enter from the bottom clear up to the middle of the height of the weir, but, with the maximum variation of flow which it was possible to obtain, there was no appreciable difference in the reading of the two piezometers, so that it is assumed that the correc-

Mr. Williams.

TABLE No. 8.—EXPERIMENTS UPON VARIOUS WEIRS FOR UNITED STATES BOARD OF ENGINEERS ON DEEP WATERWAYS AT THE HYDRAULIC LABORATORY OF CORNELL UNIVERSITY.

(1) Weir and No. of experiment.	STANDARD WEIR, 16 FT. LONG, 13.13 FT. HIGH. *h* READ 27 FT. UP STREAM.			EXPERIMENTAL WEIR, 6.56 FT. LONG. *h* READ 38 FT. UP STREAM.		
	(2) Obs. *h*. Cm.	(3) Cor. *h*. Cm.	(4) Discharge. Cu. Met.	(5) Obs. *h*. Cm.	(6) Cor. *h*. Cm.	(7) Description.
C = 3–1.	86.802	89.50	7.6030	158.006	170.00	5 to 1 up-stream slope, 8-in. flat crest; height of weir, 4.91 ft.
2.	59.205	60.10	4.1650	105.688	110.00	
3.	29.061	29.50	1.4420	51.841	52.42	
D = 4—1.	87.478	90.23	7.7040	155.159	166.45	4 to 1 up-stream slope, 8-in. flat crest; height of weir, 4.91 ft.
2.	72.707	74.16	5.7200	129.889	137.58	
3.	58.210	59.10	4.0580	103.937	108.02	
4.	43.714	44.15	2.6210	78.126	80.12	
5.	29.449	29.90	1.4715	52.260	52.85	
6.	14.583	14.81	0.5227	27.587	27.82	
E = 5—1.	87.146	89.80	7.6470	137.816	146.70	3 to 1 up-stream slope, 8-in. flat crest; height of weir, 4.90 ft.
2.	73.47	74.95	5.8155	117.268	123.00	
3.	57.78	58.59	4.0030	95.72	98.80	
4.	42.114	42.48	2.4675	74.04	75.72?	
5.	30.342	30.76	1.5340	50.02	50.65	
G = 7—1.	87.514	90.28	7.7080	134.399	142.75	2 to 1 up-stream and 2 to 1 down-stream slopes, 8-in. flat crest; height of weir, 4.895 ft.
2.	72.700	74.16	5.7200	114.005	119.20	
3.	57.998	58.84	4.032[illegible]	93.813	96.72	
4.	43.778	44.19	2.6240	72.802	74.50	
H = 8—1.	87.168	89.82	7.6500	135.558	144.00	Same as *G* with ¼-in. mesh $\frac{1}{32}$ in. thick, wire-cloth netting on up-stream slope.
2.	71.970	73.40	5.6320	115.086	120.50	
3.	57.854	58.66	4.0120	94.270	97.27	
4.	43.958	44.36	2.6420	72.636	74.35	
5.	28.804	29.20	1.4210	49.217	49.77	
I = 9—1.	87.350	89.95	7.6660	138.198	147.10	2 to 1 up-stream and 5 to 1 down-stream slope, 4-in. flat crest; height of weir, 4.94 ft.
2.	72.307	73.70	5.6675	117.366	123.00	
3.	57.702	58.51	3.9975	96.392	99.62	
4.	44.050	44.45	2.6495	74.510	76.22	
5.	29.404	29.82	1.4660	50.418	51.00	
J = 10—1.	88.406	91.19	7.8220	143.95	153.82	Vertical faces, 2.62-ft. flat crest; height of weir, 4.57 ft.
2.	73.626	75.05	5.8260	125.895	132.94	
3.	59.154	60.05	4.1610	106.66	110.98	
4.	45.168	45.58	2.7500	85.57	87.99	
5.	30.090	30.48	1.5115	60.676	61.70	
K = 11—1.	88.412	91.22	7.8300	140.010	149.15	Same as *J*, with 4-in. radius quarter-round added to up-stream corner.
2.	74.006	75.61	5.8930	121.39	127.70	
3.	58.592	59.48	4.098[illegible]	101.751	105.60	
4.	44.270	44.69	2.6690	80.511	82.52	
5.	30.540	30.90	1.5435	57.895	58.80	
L = 12—1.	72.662	74.10	5.7150	144.640	154.55	Vertical faces, 6.56-ft. flat crest; height of weir, 4.56 ft.
2.	57.89	58.71	4.0180	120.580	126.80	
3.	43.816	44.23	2.6300	93.760	96.75	
4.	29.40	29.80	1.4645	64.938	66.30	
M = 13—1.	72.632	74.08	5.7100	136.098	144.70	Same as *L*, modified as *K*.
2.	58.23	59.09	4.0550	111.67	116.60	
3.	43.71	44.16	2.6220	86.019	88.52	
4.	29.396	29.80	1.4645	59.000	60.02	
5.	14.982	15.21	0.5445	30.535	30.80	
N = 14—1.	87.22	89.90	7.6600	146.831	157.05	Rexford Flats Model; height of weir, 4.53 ft.
2.	72.59	74.00	5.7010	124.720	131.60	
3.	58.018	58.90	4.0378	101.567	105.40	
4.	43.722	44.18	2.6230	78.314	80.25	
5.	29.156	29.57	1.4470	53.549	54.25	
6.	14.442	14.66	0.5150	27.812	28.05	

TABLE No. 8—(*Continued*).

(1) Weir and No. of experiment.	Standard Weir, 16 Ft. Long, 13.13 Ft. High.			Experimental Weir, 6.56 Ft. Long.		
	(2) Obs. *h*. Cm.	(3) Cor. *h*. Cm.	(4) Discharge. Cu. Met.	(5) Obs. *h*. Cm.	(6) Cor. *h*. Cm.	(7) Description.
O = 15—1.	73.262	74.75	5.7910	122.953	129.55	Same as *N*, with rounded corner as in *K* and *M*.
2.	58.393	59.24	4.0745	101 191	105.00	
3.	43.555	44.00	2.6070	77.557	79.52	
4.	29.618	30.10	1.4848	53.041	53.65	
P = 16—1.	72.63	74.10	5.7120	12 ·.582	126.75	Little Falls Model, 3½ to 1 up-stream slope; height of weir, 4.57 ft.
2.	58.31	59.18	4.0660	98.957	102.55	
3.	43.56	44.00	2.6075	76.240	78.02	
4.	29.05	29.50	1.4425	51.514	52.00	
Q = 17—1.	72.878	74.40	5.7485	121.04	127.30	Little Falls Model;. 3 to 4 up-stream slope; height of weir, 4.57 ft.
2.	58.326	59.22	4.0725	99.763	103.32	
3.	44.29	44.72	2.6730	76.552	78.39	
4.	29.034	29.50	1.4420	51.019	51.57	
5.	18.004	18.35	0.7140	32.038	32 40	
R = 18—1.	22.900	23.30	1.0170		38.523	Indian Lake Model; height of weir, 4.65 ft.
2.	86.074	88.60	7.4835		149.362	
3.	71.992	73.40	5.6325	Correct head read on flush piezometer.	125.693	
4.	57.536	58.38	3.9810		100.766	
5.	43.314	43.75	2.5850		75.427	
S = 19—1.	14.338	14.59	0.5118		27.04	Submerged section, round crest, 1 to 1 up-stream slope; height of weir, 5.28 ft.
2.	28.435	28.85	1.3950		51.36	
3.	85.786	88.36	7.4530		142.128	
4.	70.915	72.36	5.5140		119.442	
5.	56.484	57.30	3.8710		97.858	
6.	43.812	44.38	2.6405		77.246	
7.	28.982	29.38	1.4330		53.42	

tions which were determined are probably as reliable as could be determined with the apparatus used, in which the heads were read in divisions of about 1 mm. Now, applying these corrections to Experiments Nos. 20 and 21, and applying Bazin's formula without the correction for velocity of approach, it appears that for heads on the standard weir running up to 23 cm., which means a head on the lower weir of about 1½ ft., that is, so long as the head on the lower weir was within the range of Bazin's investigations, the difference in the discharge, as given by the two weirs, is less than 2 per cent. In fact, for heads on the lower weir from a little less than 1 ft. up to about 1⅓ ft., the difference is less than 1 per cent. But, as soon as the head gets above that point the discharges depart very rapidly, the lower weir showing the higher discharge; and the variation ranges from 3% up to nearly 6 per cent.

In view of the conditions existing during the experiments it does not seem possible that the flow into the canal between the two weirs could have exceeded the leakage from it at the lower gates. In other words, it appears clearly impossible that more water passed over the lower than over the upper weir on this account; and yet, according to

Mr. Williams. the observations and formula, there is an excess of about 4% in the discharge of the lower weir, to be accounted for at the high heads. It is to be noted, however, that the observations at high heads were taken in general from high to low, and as the gauges were only read to 2-mm. divisions, it is quite possible that the canal surface may have been falling when it appeared to the observer to be stationary, and hence a somewhat greater discharge have passed the lower than the upper weir.

A portion of the excess possibly may be accounted for, that is, about 0.6 of 1%, by the narrowing of the notch of the lower weir due to the pressure of the water in the canal sides. It was contracted toward its top, slightly. The width at the top was not measured accurately. But such measurements as were made went to show that about 2 ft. from the top of the weir it was contracted nearly ½ in. This would make a difference of about 0.6 of 1% in the discharge at the higher heads. But that falls far short of accounting for all the difference. Mr. Wisner, in his discussion, has suggested that the effect of the side walls may very properly be considered to have something to do with this condition, and the extreme roughness of the sheet as it passed over the lower weir at the high heads will probably explain what remains.

In a later investigation it happened that there was obtained, incidentally, some notion of the effect of such roughness. There happened a repetition of two experiments in which the experimental weir remained in the same condition, while above the standard weir there were set additional baffles, between the two experiments, so that in the second case the water approached the weir much more smoothly than in the former. It appeared that with such roughness as existed in the standard weir, with heads of about 1½ ft. before smoothing, there was about ½ cm. more head required to deliver the same quantity of water than with the smoother approach. That is to say, if the water approaches the weir with high commotion a higher head will be required to discharge a given quantity than when the approach is smooth. The commotion at the standard weir, in the later experiments, was not to be compared with the commotion at the lower weir in the case of the high heads, in the earlier investigations. In the latter, and next to the wall, there was a roll, then came three crests and depressions, the bottom of the depressions being sometimes nearly 6 ins. below the crests. It seems, therefore, that when a proper correction is made for the effects of the roughness, the two weirs would come quite closely together. Of course, the reduction of the observations is simply applying Bazin's formula to the two weirs, *i. e.*, computing a discharge for weirs of that height according to his formula without any further corrections whatever. At the time these investigations were begun it was said by the Board of Engineers on Deep Waterways that if we could give them results within 6% of accuracy they would be abundantly satisfied.

Mr. Williams.

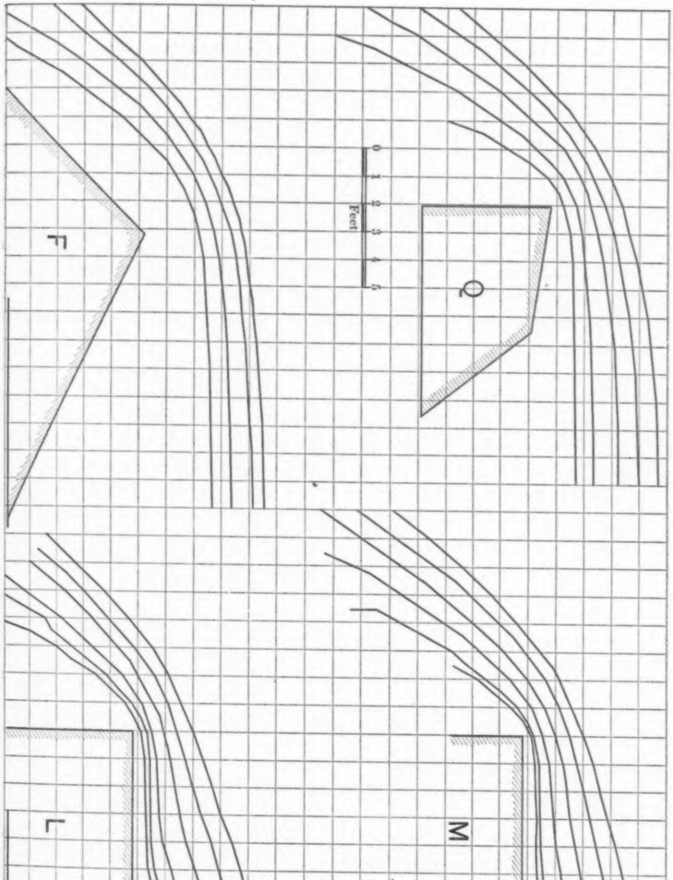


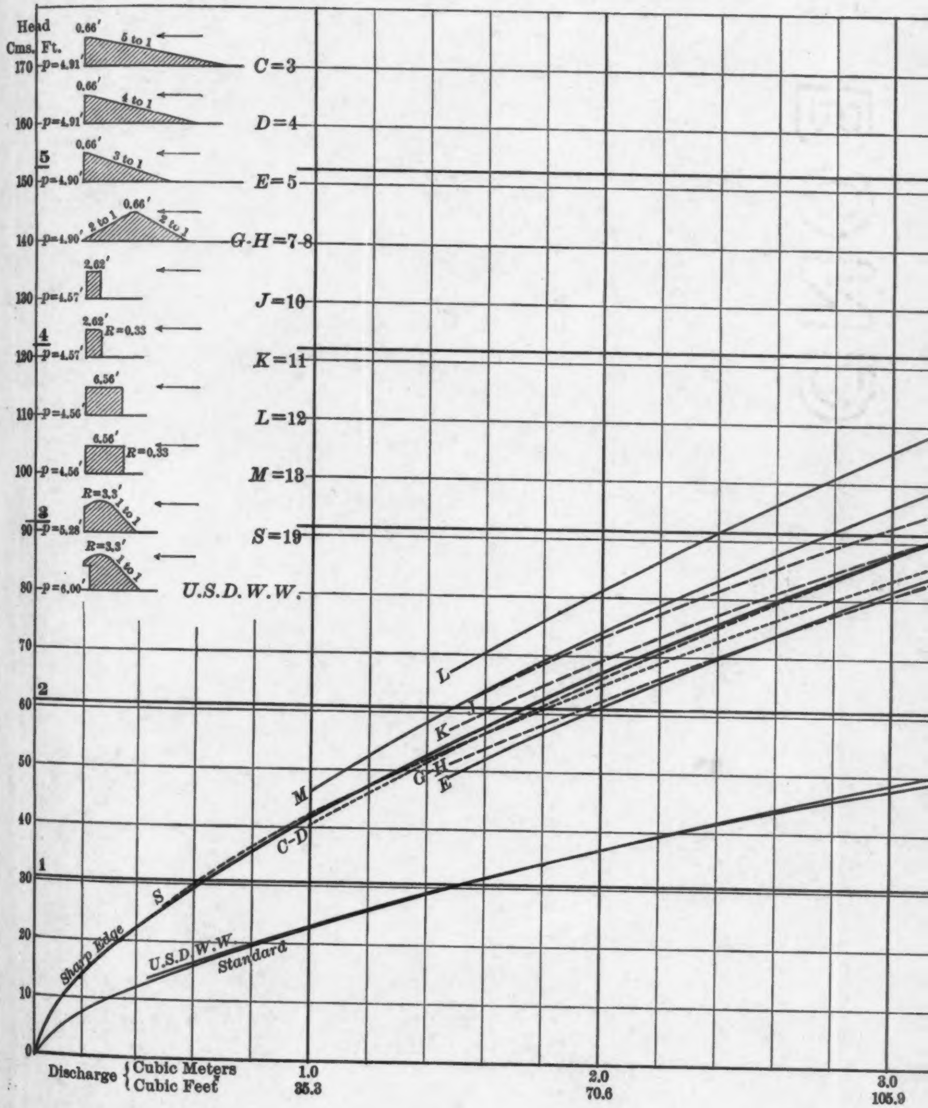
FIG. 21.

Mr. Williams. In the speaker's opinion, the results come within that range. He would not claim more. The Hydraulic Laboratory staff has performed experiments since that time, in connection with the Croton Water-shed investigations, for John R. Freeman, M. Am. Soc. C. E., which come far nearer to accuracy than 6%; but, so far as those which are given in the paper are concerned, it is very questionable if they can be depended upon within less than 6 per cent. Now, it appears to the speaker—it may be a notion in which he is peculiar—that in presenting the results of an investigation of this kind to this Society, in putting the observations upon record, forever as it were, it is most proper to present them first as nearly as possible as they were taken, to keep quite distinct the data which are facts and the data which are conclusions, to present the experiments as they were made with as little reduction as possible, so that in the future the investigator may determine for himself, in the light of such new knowledge as he may then have, just what reliability is to be put upon the observations, and what lessons are to be drawn from them.

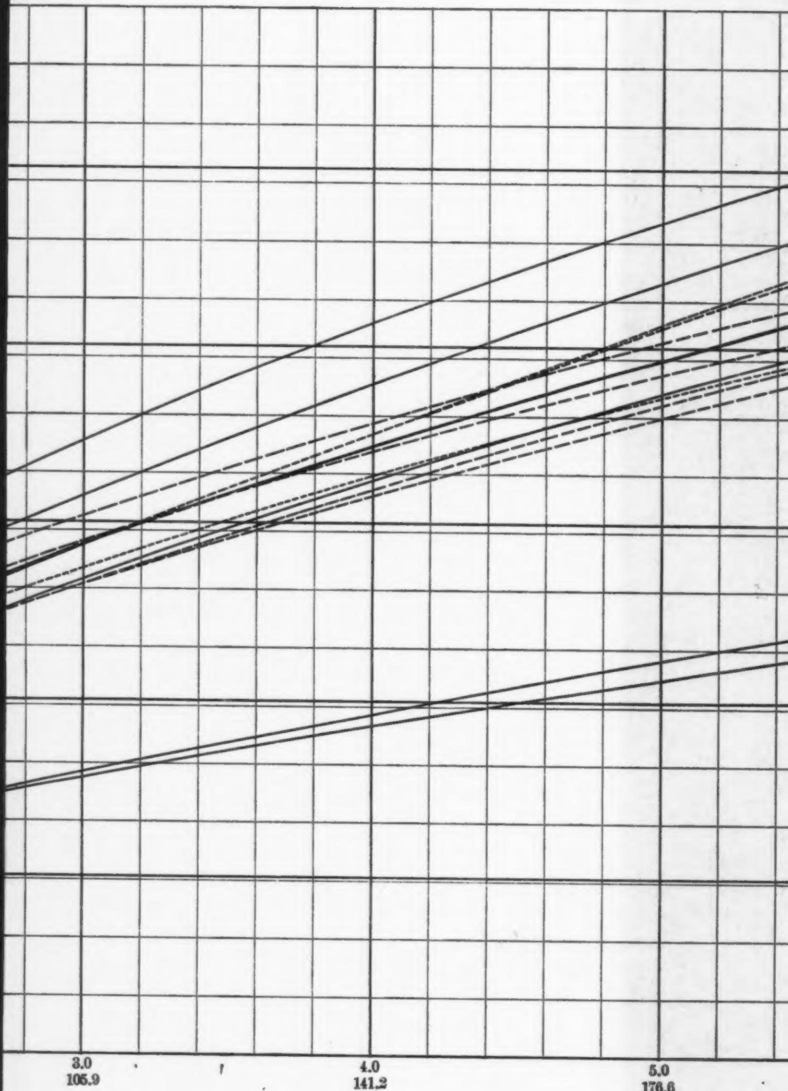
The speaker would criticise the author for having presented here a paper in which practically all is reduction, and there is no getting behind his returns, whatever we may discover in the future as to the flow over weirs. So far as the data in this paper are concerned, there is little that we can go back to and make a rigid comparison with. That which has been presented is deduced from a computed discharge of the standard weir, which has been shown to be fundamentally in error. It then follows that the whole array of coefficients and coefficient curves on pages 267 to 284, inclusive, are similarly in error and therefore correspondingly reduced in value. This error probably ranges from zero to 3 per cent.

On Fig. 4 there is the following note: "The correction for Velocity Head $\left(\frac{v^2}{2g}\right)$ as used in Reducing the Experiments is in effect Equivalent to $h_e = 1.33 \left(\frac{v^2}{2g}\right)$ for position of Piezometer 6.0 Ft. Back of Weir." Upon what authority this statement is made the speaker is unaware, but if there are any data upon which such a statement can be legitimately based it is to be regretted that the author did not give a reference thereto. So far as the Cornell experiments are concerned, there is nothing to lend support to such an assertion, and until some facts are brought to support it, it is only entitled to consideration as a rather positively expressed opinion, which, in the speaker's opinion, is contrary to fact, for the reason that we have no means of knowing how the head actually observed would compare with that which would have been observed at an opening in the side-wall, 6 ft. up stream from the crest.

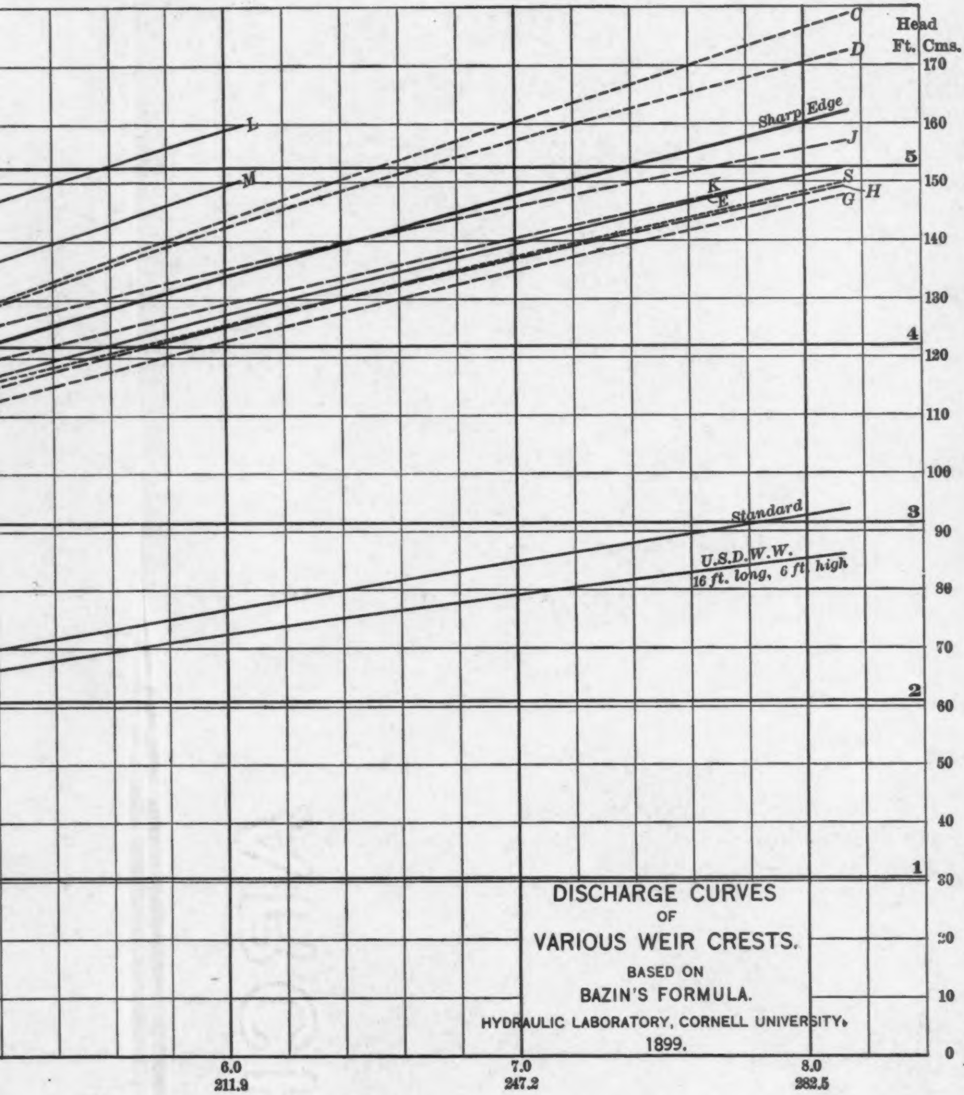
In order that the results of this investigation may be properly on



Head
Cms. Ft.
C=3
D=4
E=5
G-H=7-8
J=10
K=11
L=12
M=13
S=19
U.S.D.W.W.
0.66′
5 to 1
4 to 1
3 to 1
2 to 1
p=4.91′
p=4.90′
2.62′
p=4.57′
R=0.33
6.56′
p=4.56′
R=3.3′
1 to 1
p=5.28′
p=6.00′
Sharp Edge
U.S.D.W.W.
Standard
Discharge { Cubic Meters / Cubic Feet
1.0 35.3
2.0 70.6
3.0 105.9



3.0
105.9
4.0
141.2
5.0
176.6


Head
Ft. Cms.
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
5
4
3
2
1
C
D
Sharp Edge
J
L
M
K
E
S
H
G
Standard
U.S.D.W.W.
16 ft. long, 6 ft. high
6.0
211.9
7.0
247.2
8.0
282.5
DISCHARGE CURVES
OF
VARIOUS WEIR CRESTS.
BASED ON
BAZIN'S FORMULA.
HYDRAULIC LABORATORY, CORNELL UNIVERSITY.
1899.

record, Table No. 8 is presented, in which Column 1 gives the number of the experiment in its proper series; Column 2 gives the observed head on the standard weir by the transverse piezometer 27 ft. up stream from the weir; Column 3, this head corrected to that read by the longitudinal piezometer 27 ft. up stream from the weir and 6 ft. above the bottom of the channel of approach; Column 4, the discharge per second, in cubic meters, by Bazin's formula, Mr. Williams.

$$Q = \left[0.405 + \frac{0.003}{h}\right] \left[1 + 0.55 \left(\frac{h}{p+h}\right)^2\right] l\,h\,\sqrt{2\,g\,h},$$

where p = the height of the weir = 4.002 m. = 13.13 ft., and l = length of crest = 4.8768 m. = 16 ft.; Column 5 shows the head simultaneously observed upon the experimental weir at the up-stream transverse piezometer; and Column 6 this head reduced to the flush piezometer, or the head observed upon the flush piezometer in the bottom of the channel, 37 ft. up stream from the weir in the case of the experimental sharp edge, and 38 ft. in the cases of the other experimental weirs. All heads given are the means of those observed during the time of the experiment. These heads have been recomputed from the original field notes.

Series A, B and F (Nos. 1, 2 and 6 of the author), have been omitted, the results being too questionable to warrant insertion with the others, and E (author's No. 5) is considered as quite possibly inaccurate. The crests of the experimental weirs were approximately 2 m. = 6.56 ft. long.

While the absolute values determined by this investigation may be considerably astray, because of uncertainty, within at least 3% of the quantity of water passing the standard weir, the relative discharges of the several experimental weirs are of great interest, and on Plate XV the discharge curves of several of the types are shown, these curves being based upon that of the 16-ft. standard weir computed by Bazin's formula. These weirs were all of approximately the same height, the range being from 4.6 to 5.3 ft., so that from the plate one may readily see the effects upon the discharge caused by crests of various forms.

It will be noted that the discharge curve of the experimental sharp-edged weir divides the upper group of curves about in halves, those weirs whose curves fall above it giving a less discharge for a given head than does the sharp-edged weir.

One very interesting point is the behavior of broad flat crests. As seen by L, they give, at the lower heads, much less discharge than the standard, but, as is shown by J, and already pointed out by Messrs. Fteley and Stearns, and by Bazin, when the head reaches a point at which the sheet jumps from the up-stream edge clear or nearly clear of the down-stream corner, and the space between the sheet and crest becomes filled with eddying water, the discharge is very notably increased; so much so in the case of J that it exceeds that of the sharp-

Mr. Williams. edged weir at 4.5 ft. head. The curves *M* and *K* show the increase of discharge due to building on a 4-in. radius, quarter round, to the up-stream corner of *L* and *J*; this rounded edge adding over 11% to the discharge at a 4-ft. head in both cases.

The effects of long and short back or up-stream slopes are shown by the curves *C*, *D*, *E*, *G* and *S*. *C* being 5 to 1; *D*, 4 to 1; *E*, 3 to 1; *G*, 2 to 1, with a 2 to 1 down-stream slope added; and *S*, 1 to 1 with a 3½-ft. radius round crest. As stated by Bazin, when the inclination of the up-stream face of the weir is such as to form an obtuse angle with the bottom of the channel of approach, the tendency is to suppress the contraction of the sheet as it goes over the crest, and thereby increase the discharge, but if the up-stream slope be too gradual, the frictional resistances along it may be sufficient to counteract the gain in discharge from suppression of contraction. This appears to be the case with *C* and *D*, and, at low heads, with *S*, when the curved crest partakes of the nature of a long slope. At higher heads the 1 to 1 back-slope becomes effective and the discharge increases above that of the standard weir. From the upper curves, the weir *G* appears to have the maximum discharging capacity, but this seems to be in part due to the fact that the entry of air under the discharging sheet was restricted with it, but not with the others. The later experiments upon the United States Deep Waterways Section, 16 ft. long and shown with the 16-ft. standard weir in the two lower curves of the plate, wherein air was not admitted under the sheet, gives very nearly the same discharge at 3 ft. head, as did the weir *G*, both giving over 11% more than the sharp edge. The difference between curves *G* and *H* shows the effect of adding wire-cloth to the up-stream face of the weir *G*. Some later experiments indicate that the difference of discharge between a crest of dressed pine and one as rough as ¼-in. mesh wire-cloth will hardly amount to 3 per cent.

During this investigation, at the suggestion of Mr. George Y. Wisner, the side of the flume at the lower weir was marked off into squares, which were lettered and numbered so that the line of the surface of the approaching and discharging water upon the side of the flume could be read probably within 0.05 to 0.10 ft. During all but the first two experiments these squares were read, and from these readings eight of the most characteristic profiles of the surface curves, shown in Figs. 21 and 22, have been plotted.

Comparing *L* and *M*, the effect of the rounded corner is readily seen in the reduction of contraction at the up-stream corner, particularly at the lower heads. The influence of the back slopes is seen in comparing *D*, *F*, *Q* and *S*. It is to be regretted that the readings were not continued up stream to the beginning of the surface curvature, which, in some cases, was lost in the rapid at the throat of the flume, 48 ft. from the crests. From some of the experiments upon the effect of

Mr. Williams.

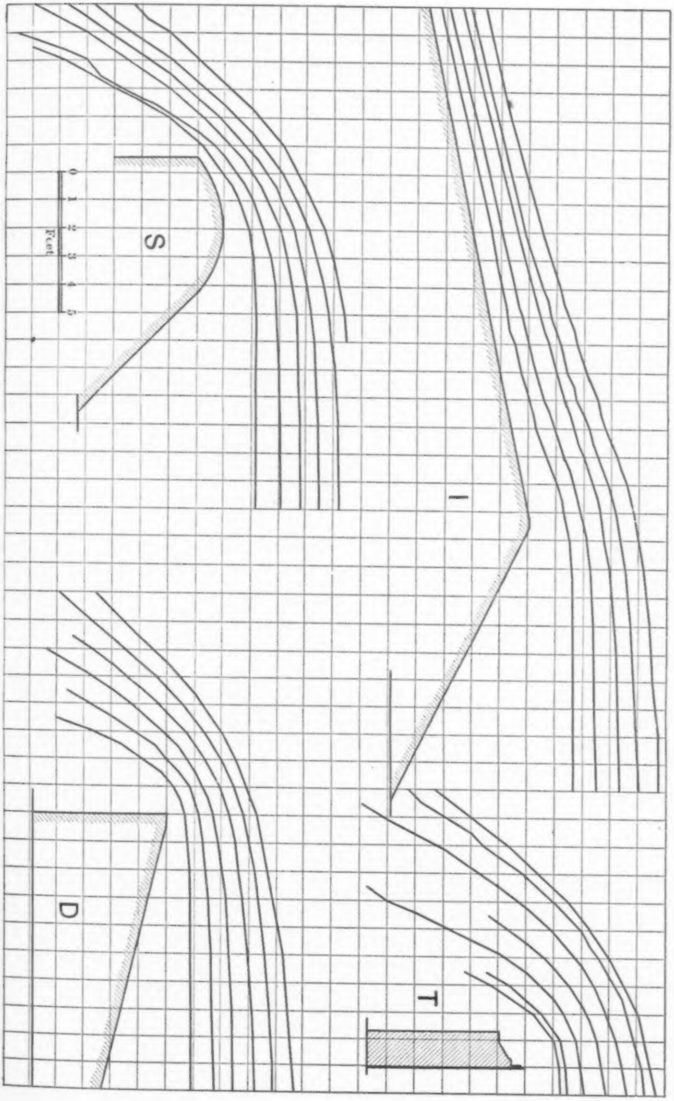


FIG. 22.

Mr. Williams. contractions in pipes it seems very probable that this contraction may have seriously affected the discharge, and in future similar experiments it would seem well to remove it much farther from the weir or nullify its effect with baffles.

The weir *I* has a peculiar discharge curve. At low heads the flow is chiefly influenced by the 2 to 1 up-stream slope giving a high discharge, but as the head increases a point is reached where apparently the slope of the apron, 5 to 1, is not sufficient to maintain the velocity necessary to free the crest, and the discharge decreases relatively to that of a sharp-crested weir, the whole weir partaking, apparently, of the nature of a broad flat crest.

For the form of crest represented in Fig. 8, the Croton experiments, upon a large-sized model quite similar to this, indicate that the discharge partakes of the nature of that of the flat crest under high heads, the water between the crests of the old and new dam reducing the friction across the top, and thus producing or permitting a discharge slightly greater than that of a sharp edge, rather than giving one 20% less, as assumed by the author by comparison with the observation on crest *L*. This point can in no way be considered as a reflection upon the judgment of the author, as these data were not available at the time he made his estimates, and the matter is only presented to indicate how far one may go astray on these questions, unless exact information has been obtained on the specific form considered.

One of the most important facts brought out in the past year's investigations in the Cornell Hydraulic Laboratory has been the formation of a vacuum more or less perfect behind the falling sheet when air is not freely admitted. With a weir 6 ft. high, the United States Deep Waterways Section, a head of 1.5 ft. has been observed to raise water behind the sheet to a height of 2 ft. above the level of the lower pool, and with a weir 8 ft. high and a 2-ft. head the water behind the sheet has stood 5 ft. above the level of the lower pool. The bottom boards of the plank aprons have been torn off frequently, apparently by the suction of the falling sheet at the toe of the dam. The possibility of a vacuum upon the down-stream face of a dam has, so far as the speaker is aware, never been considered in the design of such structures, but the pulling off of the granite facing on the down-stream side of the Austin Dam, while that at the crest remained practically intact, and other instances of similar phenomena that have been reported, seem to indicate that there may have been a very decided suction there on the occasion of its failure. This teaches that in the design of spillways, the practice of conforming them to the curve of the sheet, in order to obtain a smooth and compact overfall, should be reversed, and every precaution taken to prevent the sheet reaching the foot of the dam in a compact mass.

In conclusion, the speaker would acknowledge his great indebted-

ness for the very valuable services of his colleague, Mr. W. E. Mott, in the reduction and preparation of the data herein referred to and presented, and also in assisting in observing, under very trying conditions, during many of the later experiments. Mr. Williams.

E. Kuichling, M. Am. Soc. C. E.—This paper may fairly be regarded as a useful contribution to the literature of hydraulics, as it gives us a translation of Bazin's summary of his recent experiments on the flow of water over weirs and dams of various section, along with data obtained at the Hydraulic Laboratory of Cornell University relating to the flow over a number of dams, of both larger and different section from those used by Bazin, and under much greater heads. The method of conducting the experiments at said laboratory appears to have been similar in general to that which was followed by Bazin, except that no direct volumetric determinations of the discharge were made, as no facilities for this purpose are yet available there. On this account the calibration of the large standard reference weir could be done only by assuming the accuracy of some one of the various existing weir formulas; and, as Bazin's formula was not only the most recent one, but also purported to consider fully the influence of the height p of the standard weir above the bottom of the channel of approach, it was accordingly selected by the author as best representing the law of discharge. Mr. Kuichling.

Owing to the various algebraic errors in the paper as originally submitted, it becomes proper to state here that the correct form of Bazin's preferred formula, adapted to measures in English feet, for the flow over a sharp-crested vertical weir with end contractions suppressed, is:

$Q = m\,l\,h\sqrt{2\,g\,h}$, where the coefficient

$$m = \left\{\left(0.405 + \frac{0.00984}{h}\right)\left[1 + 0.55\left(\frac{h}{p+h}\right)^2\right]\right\},$$

and h is the observed head or depth of the water above the crest, measured at a distance of 5 m., or 16.4 ft., above the weir, while p is the height of the horizontal weir crest above the bottom of the channel, and l is the length of said crest or the width of the channel of approach, which is assumed to be of uniform section, with vertical sides and horizontal bottom. An apparatus of this description is called a standard weir when provision is made for the free admission of air under the falling sheet of water.

It should also be remembered that this formula was established from comparative experiments with standard weirs of different length and height, within a rather limited range and with comparatively low heads. Bazin's largest weir was only 6.56 ft. long and 3.72 ft. high, on which a maximum head of about 1.0 ft. was used, on account of restricted volumetric capacity. For heads from 1.00 to 1.32 ft. it was necessary for him to reduce the length of this weir one-half, or to

Mr Kuichling. 3.28 ft.; and for heads from 1.32 to 1.77 ft., the latter length was again reduced one-half, or to 1.64 ft., while the height was reduced to 3.30 ft. In reference to the experiments made with the last-mentioned weir, Bazin states that as it was possible for the retardation by friction against the side walls to become appreciable in so small a weir or channel, the observations made therewith were used by him only to deduce the law of increase of the coefficient m for heads greater than 1.32 ft., whereas the more numerous and precise results furnished by the two longer weirs served as the basis for determining the discharge of the standard weir. The computed values of the coefficient m for said longer weirs under the same heads up to 1.00 ft. were found to agree quite closely, whence Bazin inferred that for moderate values of h the law of the discharge $q = \frac{Q}{l}$ per unit of length of the weir was not materially affected by the length l, provided that the same was more than 3.0 ft., and that the curve obtained by plotting the values of m as ordinates to the corresponding values of h as abscissas could fairly be extended from $h = 1.00$ ft. to $h = 2.0$ ft., governed somewhat in shape by the curve for m given by the shortest weir.

Furthermore, to find the effect of varying the height p of a standard weir, Bazin next compared the heads h for known discharges Q over his longest weir, having $l = 6.56$ ft. and $p = 3.72$ ft., with those which were observed for the same discharges over four secondary standard weirs of the same length l, but having heights of 2.46, 1.64, 1.15 and 0.78 ft.; and he also computed and plotted the values of m for each series of experiments corresponding to the different values of h. The latter, however, did not exceed $h = 1.48$ ft. on the standard reference weir having $p = 3.72$ ft. Within these limits of head and height, and the same length l, it was found that the values of the coefficient m derived from experiment did not vary more than 1% from those given by the aforesaid expression:

$$m = \left\{ \left(0.405 + \frac{0.00984}{h}\right) \left[1 + 0.55 \left(\frac{h}{p+h}\right)^2\right] \right\}$$

except in the case of the lowest weir, were the value of m for small heads given by the formula is about 2% more than found by experiment, although such excess gradually reduces to zero as the head h increases to 1.30 ft.

Such is the foundation of Bazin's formula for the discharge over standard weirs with suppressed end contractions and heads h measured 16.4 ft. above the crest, and the question now arises whether it can be applied without material error, either to the large standard weir at the Hydraulic Laboratory of Cornell University, where the length $l = 16.0$ ft., the height $p = 13.13$ ft., and the heads h, up to a maximum of 3.0 ft., were measured 10 ft. above the crest, or to

the author's secondary standard weir at the same place, where the length $l = 6.53$ ft., the height $p = 5.26$ ft., and the heads h, up to a maximum of 4.874 ft., were measured 37 ft. above the crest. Mr. Kuichling.

In the absence of direct experimental evidence, little further can be said on this subject except to quote the remark of the late Hamilton Smith, Jr., M. Am. Soc. C. E., made on page 169 of his classic treatise on hydraulics, in concluding his comparison of the values of the coefficient of discharge for weirs of different length and large heads, that "such speculations, though possibly ingenious and plausible, when tested with facts, generally prove to be very wide of the truth." While this remark was intended to apply to cases far beyond the limits of accurate experiment, it is, nevertheless, also pertinent in some degree where such limits have been considerably exceeded, as in the present case; and hence it will be prudent to suspend judgment about the accuracy of the large discharges until the figures are properly corroborated. Under these circumstances, accordingly it, appears admissible to indulge in a little speculation about Bazin's formula for the coefficient m.

In this formula both the observed head h and the height p of the weir above the bottom of the uniform channel of approach are taken into account, but the width of the channel or the length l of the weir is omitted for the reason already given. Now, while the retardation of the flow by friction on the sides of the channel in the vicinity of the weir may be negligible in the case of long weirs and small heads, or up to a certain limiting ratio $\frac{h}{l}$, it is evident from observations of the relative velocities of thick sheets of water that beyond such limit this lateral friction cannot be disregarded; and hence a rational formula of general applicability should make the value of the coefficient m dependent also upon the channel width or weir length l, thereby allowing the formula for the discharge over a standard weir to pass gradually with reducing value of the height p into that for a uniform channel. This condition is not embraced in Bazin's formula, as will be seen from the following consideration.

When the height p becomes very small relatively to the head h, or disappears entirely, the above formula for m becomes practically $m = 1.55\left(0.405 + \frac{0.00984}{h}\right) = a + \frac{b}{h}$, while that for the discharge Q becomes: $Q = m\ l\ h\sqrt{2\,g\,h} = \sqrt{2\,g}\left(a + \frac{b}{h}\right) l\,h\sqrt{h}$, which should obviously correspond with the formula for the flow in a uniform open channel. Taking for comparison in this case the Chezy formula on account of its simplicity, we will have for the same discharge: $Q = A\,v = A\,c\sqrt{r\,s}$; and by replacing the section A by its equiva-

Mr. Kuichling. lent $l\,h$ in the nomenclature of the weir formula, and the hydraulic radius r by its equivalent $\frac{l\,h}{l+2\,h}$, there follows:

$$Q = c\sqrt{\frac{l\,s}{l+2\,h}} \cdot l\,h\sqrt{h}$$

By equating these two values of Q, which are manifestly alike, we obtain: $\sqrt{2\,g}\left(a+\frac{b}{h}\right) = c\sqrt{\frac{l\,s}{l+2\,h}}$, which is an entirely incongruous result, being true only for particular values of l, h and s when the value of c is given. The inference is, therefore, justifiable that Bazin's formula can only give correct results in the neighborhood of the experimental limits from which it was derived, and that discordant results will probably be found if it is applied to conditions considerably beyond said limits.

TABLE No. 9.

For observed heads on weir of:	$h =$	1.00	2.00	3.00 ft.
1. Author's values of coefficient m for the experimental standard weir 6.56 ft. long and 5.26 ft. high, are, as per page 295............	$m =$	0.4174	0.4106	0.4094
2. Values of m computed by writer from Bazin's formula for same weir, are..................	$m =$	0.4207	0.4270	0.4379
3. Values of m deduced by writer from Bazin's table in Annales des Ponts et Chaussées, 1888, page 446, for a standard weir 1.60 m., or 5.26 ft. high, are............	$m =$	0.4232	0.4252	Beyond limit of Table.
4. Values of m computed by writer from Bazin's formula with p = 13.13 ft., corresponding to large standard weir at Cornell University Laboratory, are.................	$m =$	0.4160	0.4139	0.4160

It may also be mentioned that Bazin's expression for m attains a minimum value for a particular value of h with each value of p. By applying the usual mathematical process for maxima and minima to said expression it will be found that for Bazin's standard weir with $p = 3.723$ ft., m will become a minimum $= 0.4245$ for $h = 0.814$ ft.; for the author's experimental standard weir at Cornell University, with $p = 5.26$ ft. m will become a minimum $= 0.4207$ for $h = 1.003$ ft.; and for the large standard weir at the same place m will become a minimum $= 0.4137$ for $h = 1.765$ ft. On comparing these latter values of m, as well as a few other computed values of m for $h =$ 1.0, 2.0 and 3.0 ft., with those given by the author in Fig. 2, a considerable disagreement will be noticed, whence a numerical error has probably been made in the author's computations. The text also indicates that the said values of m in Fig. 2 refer to the experimental standard weir with $l = 6.53$ ft. and $h = 5.26$ ft.: but on exam-

ining the diagram of discharge of the large standard weir with l = 16.00 ft. and p = 13.13 ft., in Fig. 4, it will be found that the same values of m, or $\frac{Q}{l}$, were used as given in Fig. 2 for the smaller weir; and if such be the case another slight error has been made in the paper. The discrepancies in the values of m just mentioned are exhibited in Table No. 9. Mr. Kuichling.

The use of the author's experimental sharp-crested weir, 6.53 ft. long and 5.26 ft high, has not been made clear in the text. Apparently, it was built for the purpose of calibrating the large standard weir, 16.00 ft. long and 13.13 ft. high, but as the author states explicitly on page 292 that "for the reduction of Experiments Nos. 20 and 21 (referring to said two weirs) a discharge curve has been computed for the upper (large) weir for heads up to 0.6 m. (1.969 ft.), using Bazin's formula," it is not easy to discover its significance unless to find therefrom the comparative values of m for large heads measured at a distance of 37 ft. above its crest. In this event the experimental data would have been of much interest, as they would have established in some degree the truth or error of Bazin's formula when applied to high heads, and it is to be hoped that the author will supplement his paper with such data.

Another point of interest is the matter of the proper distance above the weir or dam at which the head h should be measured, as well as the details of such measurement; but as this part of the subject has been fully considered in the discussion by Gardner S. Williams, M. Am. Soc. C. E., further reference thereto may here be omitted.

E. A. Fuertes, M. Am. Soc. C. E.—The remarks of Mr. Williams, upon the lack of methods for performing volumetric measurements at the Hydraulic Laboratory of Cornell University, are pertinent and justifiable; but it must be borne in mind that the laboratory is as yet unfinished. There are, indeed, methods available, on the spot, for more or less accurate and direct volume measurements, but not of the nature intended to be made in the future, and for which, fortunately, topographical conditions lend themselves admirably to measure almost any desired volume of water. So far, this laboratory is a huge engine, with possibilities. Its final suitableness is a question of time and money. The speaker wishes to impress upon engineers the fact that such a laboratory exists; that its resources can be extended unlimitedly, and that one of the methods which will bring us quickly to the desirable conditions of which Mr. Kuichling speaks is to enlist the good offices, the good-will and the aid of the hydraulicians of this country to send us problems to solve, either under our own direction or under the direction of such visiting engineers as may desire to superintend the work. Mr. Fuertes.

It seems advisable to say, however, that while Cornell University will do everything that may be possible with its personnel, equipment

Mr. Fuertes. and other facilities for the progress of hydraulic science, it seems reasonable that such expense as may be necessary and may not be covered by the facilities of the laboratory at the time should be defrayed by the party seeking information. Nothing will be stinted on the part of Cornell University.

Mr. Horton. ROBERT E. HORTON, Esq. (by letter).—The results of the Cornell experiments and those of Bazin emphasize primarily the necessity of using a variable coefficient in the weir formula, whenever the flow over an irregular-crested dam is to be computed. To facilitate calculations of this character, and thereby make the results of these experiments more readily applicable in practice, the accompanying diagram, Plate XVI, has been prepared. Having selected the value of the coefficient *C*, required for the given head and section, its value is scaled off on the line of ordinates at the left of the diagram and traced horizontally to its intersection with the transversal corresponding to the head required; the projection of this point of intersection on the line of abscissas will show the flow over the dam in cubic feet per second for one lineal foot of crest, without lateral contractions.

The chief distinction between the experiments made at Cornell University and those of previous experimenters, including Bazin, lies in the fact that the former were made on "life size" dams, as it were, and extended through a much wider range of heads. This is illustrated by Table No. 10, which shows the superior limit of various experiments on flow over sharp-crested weirs.

TABLE No. 10.

Name of experimenter.	Date.	Limiting head used in experiments, in feet.	Length of crest of experimental weir, in feet.	Distance up stream from weir to point of measuring *H*, in feet.
Poncelet and Lesbros..	1828	0.682	0.66	11.48
Lesbros..............	1834	0.8009	0.6562	11.48
Francis..............	1852	1.62	9.995	6.00
Hamilton Smith, Jr...	1874–76	1.7327	2.586	7.6
Fteley and Stearns....	1878	1.6038	19	6.0
Bazin................	1886	1.77	6.56	16.40
Cornell experiments...	1899	4.672	6.56	38.0

Owing to the limitations pointed out by the author, the Cornell experiments were less elaborate as regards minuteness of detail than were those of Bazin; as a rule, only four or five experimental coefficients were determined as a basis for plotting the mean coefficient curves. Bazin usually obtained twenty or more experimental coefficients, within a much narrower range of variation of head. The advantage of repeating the experiments for small increments of head will not be as

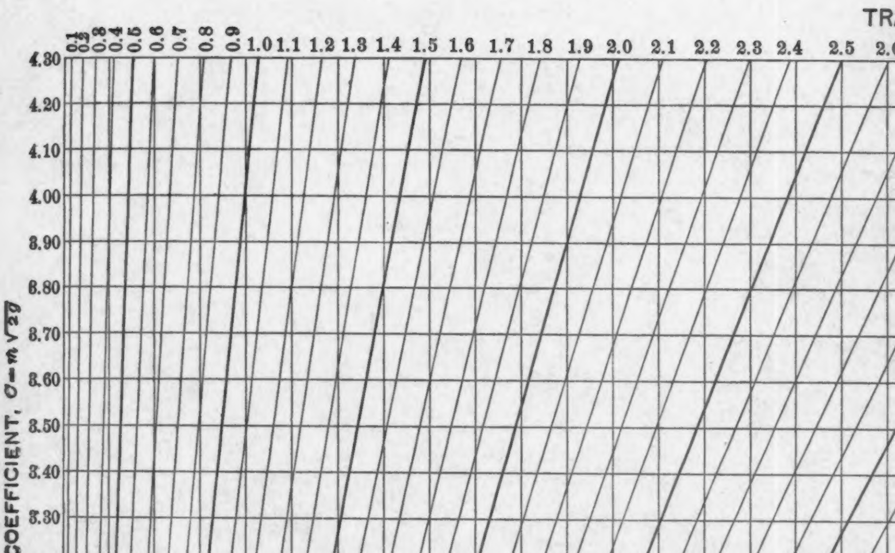
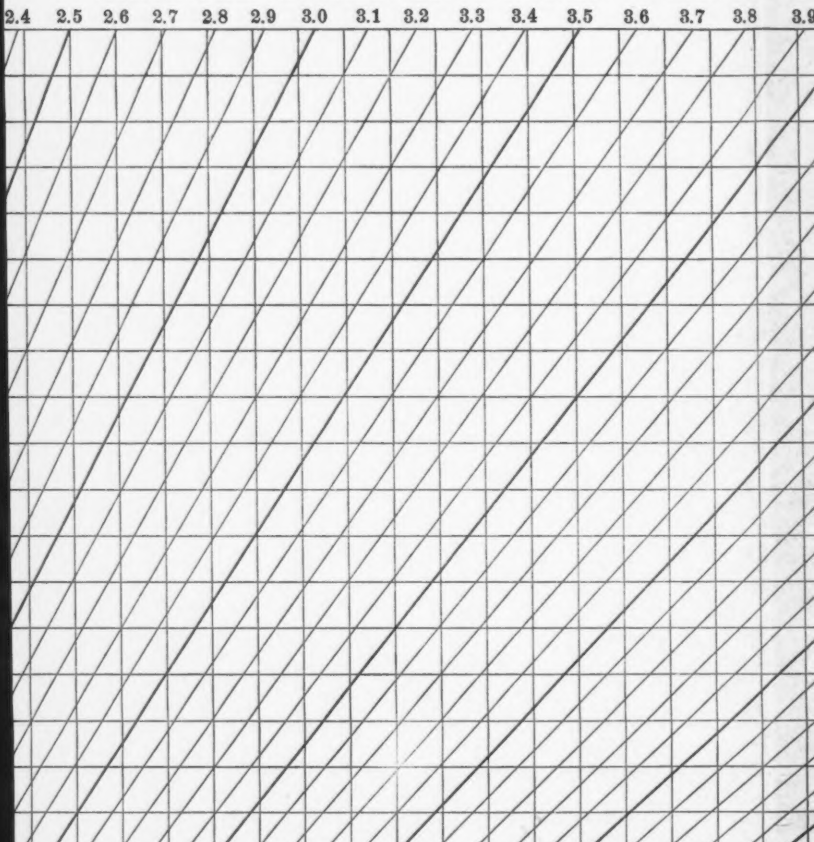

TR
VALUE OF THE COEFFICIENT, $C = m\sqrt{2g}$
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5 2.6
4.30 4.20 4.10 4.00 3.90 3.80 3.70 3.60 3.50 3.40 3.30 3.20 3.10 3.00 2.90 2.80 2.70 2.60 2.50 2.40 2.30
0. 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18.

DIAGRAM SHOWING FLO

$Q = CLH^{3/2}$

TRANSVERSALS REPRESENT DEPTH ON CREST OF WEIR IN FEET

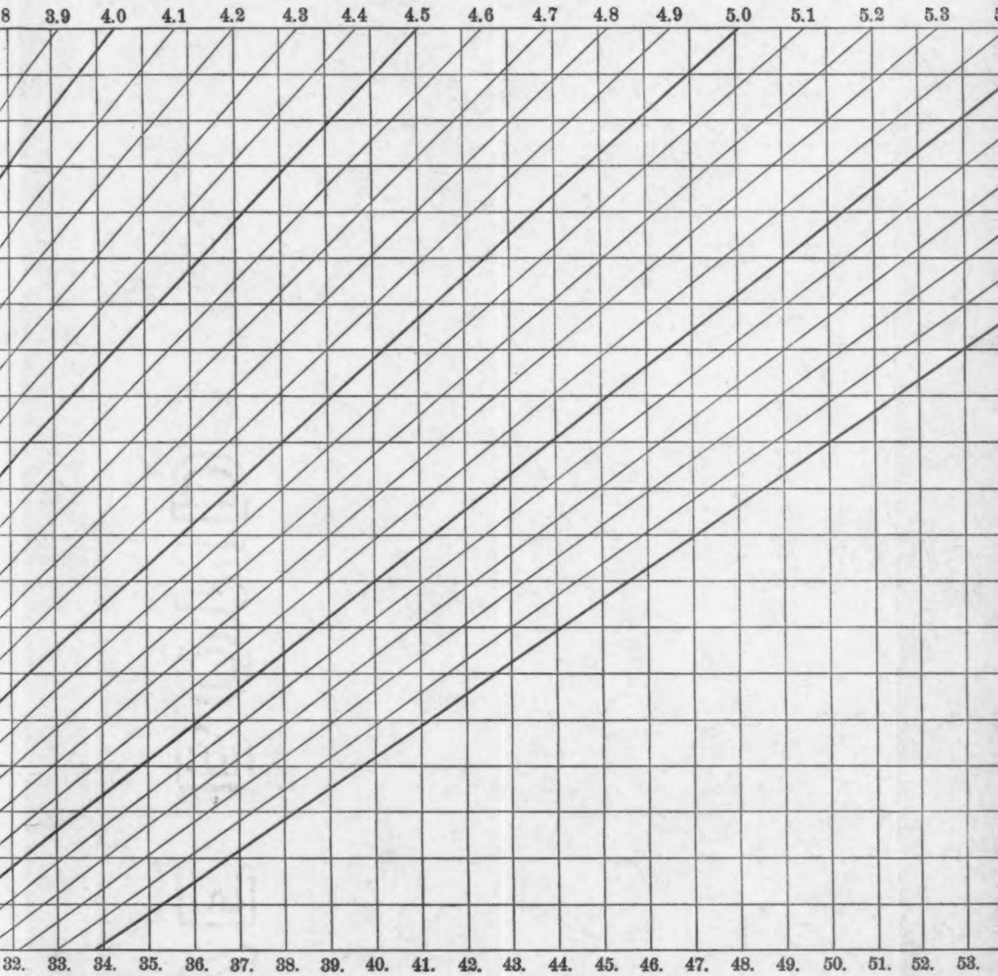


Q=FLOW IN CUBIC FEET PER SECOND, PE

FLOW OVER DAMS,

$.H^{3/2}$

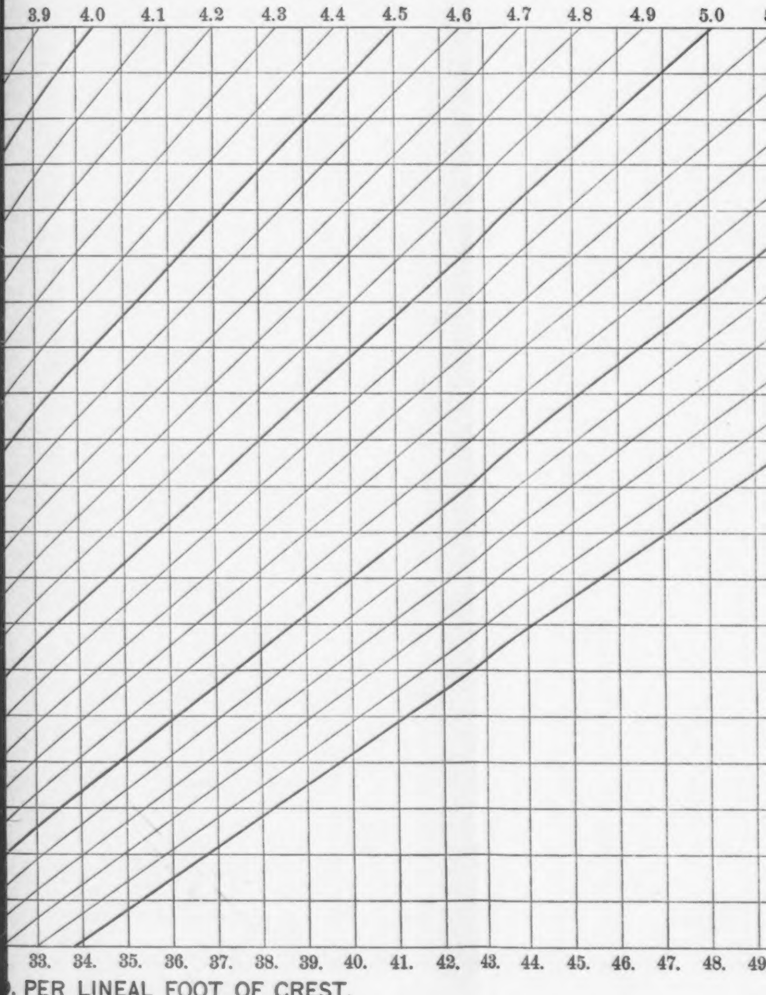
N FEET, MEASURED TO THE SURFACE OF STILL WATER.



ND, PER LINEAL FOOT OF CREST.

LOW OVER DAMS,

FEET, MEASURED TO THE SURFACE OF STILL WATER.



, PER LINEAL FOOT OF CREST.

PLATE XVI.
TRANS. AM. SOC. CIV. ENGRS
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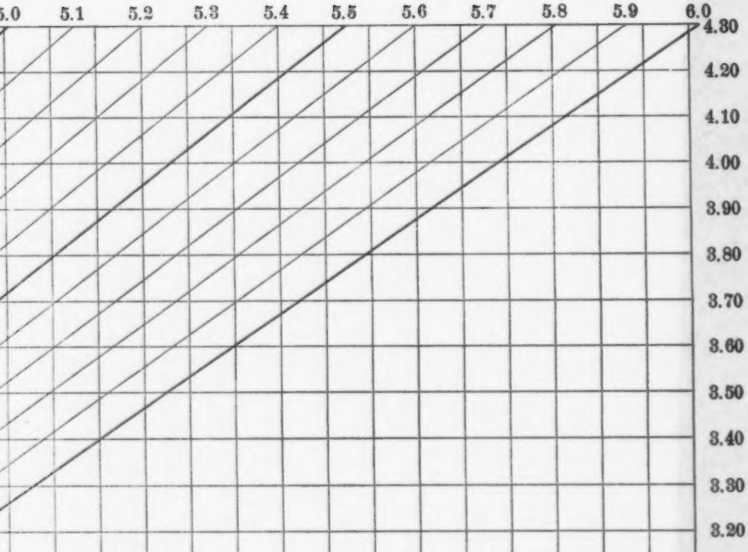
HORTON ON FLOW OF WATER OVER DAMS.



Mr. Horton.

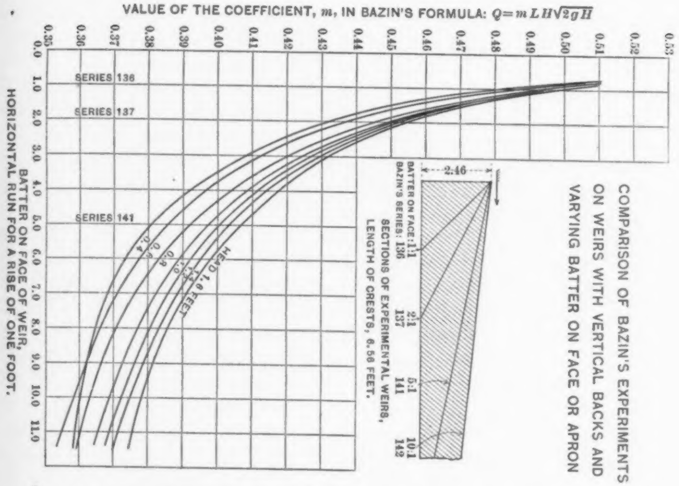


FIG. 23.

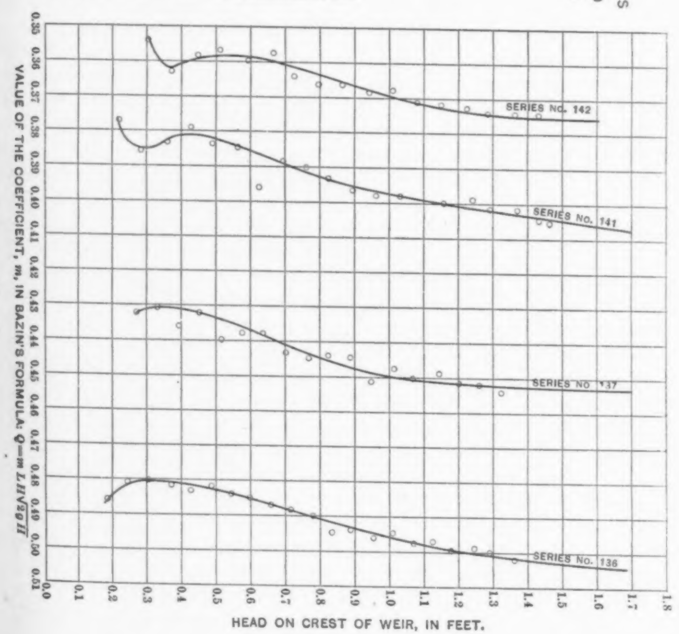


FIG. 24.

Mr. Horton. great, however, as might at first be imagined, in cases where the species of nappe does not change. In accordance with a well-known law of Least Squares, the precision of the mean coefficient curve will be proportional to the square root of the number of observations it represents. Quadrupling the number for each section would result in dividing the numerical factor for precision by two; moreover, the coefficient for each period in the Cornell University experiments is deduced from the mean of a relatively large number of observed heads on the weirs. This number, being more than twenty in each case, is so great that any moderate increase in the number would not have resulted in a proportional gain in precision. An examination of the coefficient curves will show that, as regards the individual coefficients, they appear no more discordant when plotted than do those of Bazin.

In the design of dams with depths sometimes as great as 12 ft. on their crests, there is a pressing need for data which will apply with a reasonable degree of accuracy. It is to be hoped that the Cornell experiments may be extended by the determination of additional coefficients for intermediate heads, and by experiments on a wider variety of dam sections than was possible at the time these experiments were made. Hitherto, engineers have generally accepted the standard sharp-crested weir as being the most reliable means of measuring relatively small volumes of flowing water. A free nappe, which is neither depressed, adherent, nor wetted underneath, represents the ideal condition, and might be termed a "Standard Nappe." This condition, as has been pointed out by Bazin, is very difficult of attainment. Most of the modifications to which the nappe is subjected under different conditions are phenomena belonging to comparatively low heads; for many, if not all, forms of weir sections, the nappe undergoes these various metamorphoses with heads not exceeding from 1 to 2 ft., and then settles down to a stable form which is retained for all higher heads. These facts point to the conclusion that weirs or dams are best adapted for the measurement of large streams of water forming depths of several feet on their crests.

Francis did not experiment with depths of flow exceeding 1.6 ft., although he computed discharge tables, by means of his formula, for heads up to 3 ft., and, as pointed out by the author, the constant coefficient, 3.33, has commonly been applied for all heads, however great. The formula of Bazin furnishes, perhaps, the most reliable means of determining the flow over low weirs with slight heads, and, in reducing the Cornell experiments, it has been used as a basis for determining the flow over the standard sharp-crested weir. Referring to Fig. 2, it will be seen that, for heads from 1 to 6 ft., the discharge coefficients deduced from the Cornell experiments Nos. 20 and 21, for a standard sharp-crested weir, remain nearly constant, varying only between the limits 3.350 and 3.284. The average value is 3.306, or

Mr. Horton.

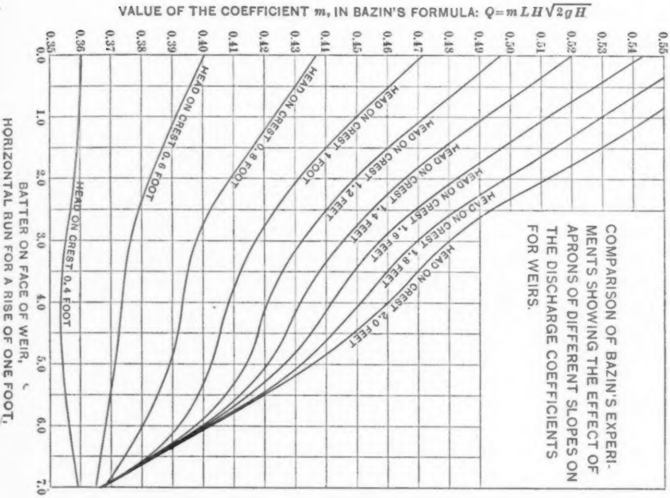


FIG. 25.

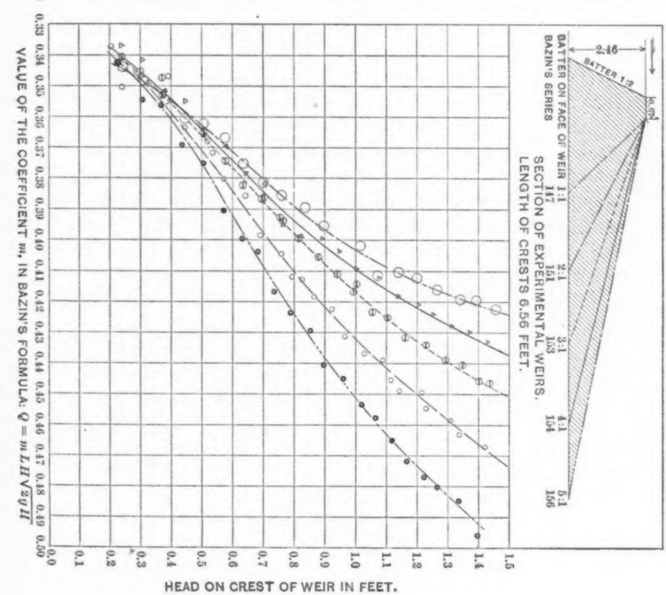


FIG. 26.

Mr. Horton. 0.7% less than the constant value 3.33, assumed by Francis, from a series of experimental coefficients varying from 3.31 to 3.35. Francis' formula, therefore, is practically verified for heads up to the limit of the Cornell experiments, and its reasonable use, for any ordinary head on a sharp-crested weir, appears to be justified.

It is greatly to be regretted that Bazin's experiments were not conducted on such a scale as to make his coefficients legitimately applicable to American mill-dams. This they certainly are not, and, while they may serve a useful purpose in measuring the flow of water in small modules, as in the partition of streams for irrigation purposes in the Western States, yet their chief interest to American engineers lies in the large number of facts which Bazin brought conspicuously before our attention, regarding the physical phenomena accompanying flow over weirs. The value of these results, qualitatively rather than quantitatively, can hardly be overestimated.

In this connection, one or two points may perhaps be brought out more strongly than has been done by Bazin in publishing his original researches. In order to show the effect of aprons of varying batter or slope, on the discharge coefficient, Bazin's Series, Nos. 136, 137, 141 and 142 have been selected. The experimental coefficients for each series have been plotted, and curves have been drawn, intended to fairly represent the experiments, as shown in Fig. 24. In each of these experiments the nappe appears to have adhered to the crest throughout all but the very lowest heads, for which there may have been a partial vacuum underneath the nappe, producing the short turn in the base of the first and second coefficient curves. In each case the coefficients increased gradually with the head, indicating that the effect of the friction of the long flat apron diminished in proportion as the head increased. This illustrates a general statement which may be made that, as the head increases, the influence of peculiarities in the form of the weir section, in modifying the discharge coefficient, becomes less and less significant, and, under excessively high heads, the coefficient curves for all cross-sections will tend to converge.

A series of coefficients for various heads, taken from Fig. 24, has been plotted in a slightly different form in Fig. 23, using the coefficients as ordinates, and the slope or batter on the down-stream face as abscissas. This brings prominently to our notice the greatly diminished discharging capacity of a dam with a flat apron, as well as the tendency of all the curves to become tangent to a common asymptote under high heads.

Comparing, in the same manner as before, Bazin's Series Nos. 147, 151, 153, 154 and 156, we obtain the series of coefficient curves shown in Fig. 26. The cross-sections for these series are precisely like those shown in Figs. 23 and 24, except that, instead of having a vertical

Mr. Horton.

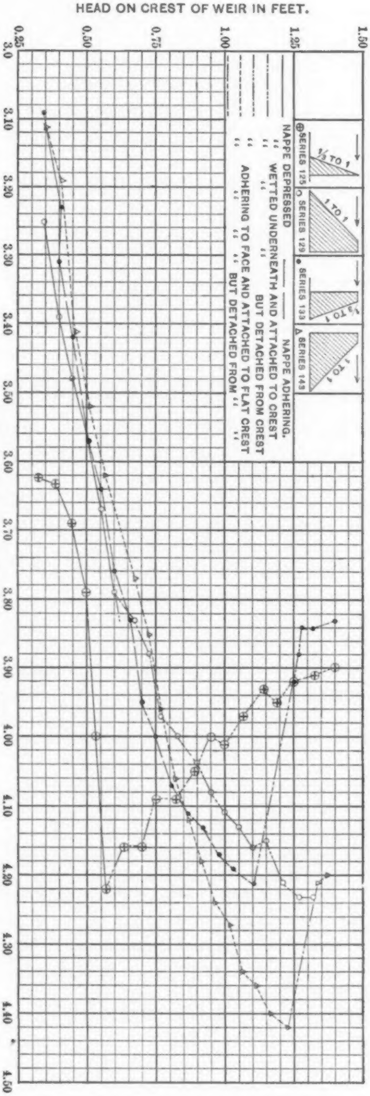


FIG. 27.

Mr. Horton. back, the up-stream face has a batter of 1 horizontal to 2 vertical, and a flat crest 0.67 ft. in width. The forms of the coefficient curves and the characteristics of discharge appear to be completely modified. Starting with nearly the same coefficients for all series at low heads, the coefficient curves diverge rapidly to the limit of the experiments. For low heads the influence of the apron is completely masked by the effect of the broad flat crest.

A number of similar comparisons might be made from Bazin's experiments, but enough has been given to show that no general formula for flow over a dam of irregular section, using the slopes of the faces and the breadth of the crest as independent variables, can be deduced; the coefficient curve for each combination must be determined separately, although the way in which any single element tends to modify the discharge may readily be pointed out.

In all the series of experiments above discussed, the nappe retained a sensibly stable form throughout the entire range of the experiment, being attached to the flat surface of the apron in each instance. The coefficient curves are continuous smooth lines. Where the nappe becomes depressed, detached or wetted underneath, during the progress of the experiment, the resulting coefficient curve, mathematically considered, will be a series of discontinuous arcs, possibly disconnected, and terminating abruptly in *points d'arret*, whenever the form of the nappe changes. Four series of coefficients, as deduced by Bazin, for weirs with variable nappes, are shown in Fig. 27. The condition of the nappe in each instance is indicated on the diagram. For dam sections like these the nappe may assume four distinct forms as regards its behavior with reference to the down-stream face of the weir, and, in addition, it may either adhere to, or be detached from, the crest. Either of these conditions may occur in combination with any of the four previously mentioned, so that by simple permutation we find that eight series of discharge coefficients are possible for a single form of section. On the other hand, there are many forms of weirs, such, for example, as that used in Cornell experiment No. 19, for which the nappe adheres to the crest and apron under all heads, and is subject to no modifications whatever. Owing to the permanence of the nappe form, such a section possesses some advantages over a sharp-crested weir for use as a standard section in making weir measurements.

Mr. Parmley. WALTER C. PARMLEY, M. Am. Soc. C. E. (by letter.)—This paper is a welcome addition to the literature on weir flow. About all that has been known of this subject up to the present time is what has been given to us in the work of Francis, Fteley and Stearns, and Bazin. Their experiments cover a rather narrow range of conditions, and hence the experiments carried out at Cornell make a substantial addition to our knowledge of the subject.

In the method of measuring the head upon the weir, Mr. Rafter has followed in general the processes used by the other experimenters, although in details they are varied. The height above the crest of the weir is not taken by direct observation upon the water surface, but by observations of the water surface in glass gauges at the side of the channel. As a consequence, tedious and involved corrections must be applied to the readings obtained from one set of piezometers in order to make them check with the results of the others. This method introduces uncertainty as to the proper place in the channel of approach of placing the inlet tubing, the manner of access to it, and the possibility of eddies vitiating the results. It is doubtful if holes bored in round tubing will give a correct record of the head, inasmuch Mr. Parmley.

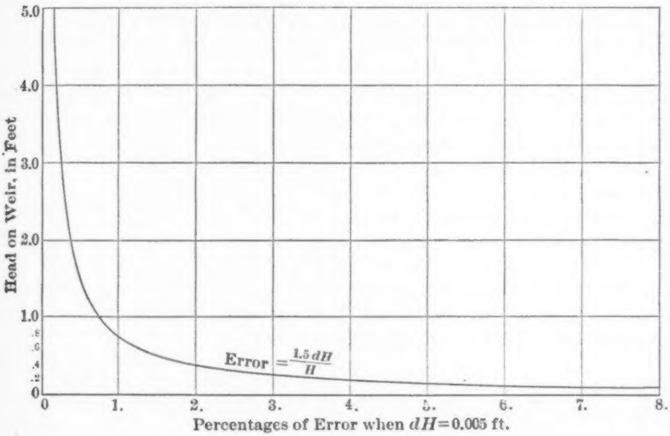


FIG. 28.

as the orifice is in a curved surface, when theory and experiment agree that the orifice should be at right angles to the motion of the water and through a plane surface parallel therewith. The experiments of Fteley and Stearns showed that the pressure near the up-stream side of the weir varied materially from the bottom to the top of the channel and with the distance from the weir. It is necessary, therefore, that the tubing enter the channel up-stream from this region of erratic pressures and, in a given case, it is not altogether certain just where these variations of pressure cease. The author does not explain whether or not any correction for capillary action in the piezometer tube was made, nor the amount, if such correction was made.

It seems to the writer that the method of Bazin is preferable. He provided recesses at the side of the channel where the actual surface

Mr. Parmley. of the water could be measured with a hook gauge. It undoubtedly has the advantage of avoiding the uncertainties mentioned. The author states that the readings of the piezometers were taken to 2 mm., that is, to within about 0.0066 ft. While an error of this amount is of no consequence when large heads are being measured, it becomes of considerable importance if the heads are small.

From his experience with many weirs under varying conditions, the writer has found the use of a steel tape graduated to hundredths of a foot very satisfactory. A sharp-pointed plumb-bob is attached to the end of the tape by means of a wire hook. The measurement to the surface of the water is taken from some well-defined point conveniently located above the water surface. These readings are made to thousandths, and in ordinary work can be made with certainty to within 0.005 ft., which is closer than the readings by the author's piezometers.

In order to illustrate the importance of accurate measurements when small heads are upon the weir and the relative unimportance of such accuracy for large heads, let us take the ordinary Francis formula, considering only a length of 1 ft. of weir.

$$Q = C\,H^{\frac{3}{2}}$$

Differentiate, and we have:

$$d\,Q = \tfrac{3}{2}\,C\,H^{\frac{1}{2}}\,d\,H$$

Now, the error of any gauging when $H + d\,H$ is taken as the head, instead of the true head H, will be the quantity dQ, and the ratio of the error to the true quantity Q, will be,

$$\frac{d\,Q}{Q} = \frac{3\,C\,H^{\frac{1}{2}}\,d\,H}{2\,C\,H^{\frac{3}{2}}} = \frac{1.5\,d\,H}{H}$$

This formula gives the exact error if $d\,H$ is an infinitessimal, and a nearly correct value of the error if $d\,H$ is some very small quantity. Assuming $d\,H = 0.005$ ft. the writer has calculated the error for different heads upon the weir. The results are shown in Fig. 28, and are expressed as percentages of the true volume flowing. The heads are on the left and the corresponding errors for the various heads at the bottom of the diagram. It will be observed that if the head is 0.1 ft., the error is 7½%; if the head is 1 ft., the error is 0.75%, and if the head is 5 ft., the error is only 0.15 per cent. It seems to the writer, therefore, that the required degree of precision could have been attained, and with greater certainty as to the final results, if a simpler and more direct method of measuring the heads had been followed.

It is probable, also, that some of the irregularities in the resulting coefficients are due to the method of observing the head. There is no

apparent reason why the coefficients of a standard sharp-crested weir should increase both ways from a minimum as shown in Fig. 7, and it is probable that the same cause accounts for the peculiarities in the coefficients of Experiments Nos. 1, 5, 12 and 16, whatever that cause may have been. Mr. Parmley.

The writer, agreeing with Professor Williams, is unable to understand the author's use of Bazin's formula on page 292.* After having correctly found the volume by the formula of Bazin, he apparently makes a further correction for velocity of approach, on pages 293 and 294. Even if such correction were necessary, the reason is not clear why the correction taken was $\frac{v^2}{2g}$, instead of $1.33 \frac{v^2}{2g}$, given as the average value by Fteley and Stearns. If the writer understands Bazin's formula, no correction for velocity of approach is necessary, since that is taken account of already in the formula and the coefficients given in his table.

This suggests what seems to be an obscure point in the discussion of weirs in most text books on hydraulics. In order to use correctly any weir formula it is absolutely necessary to use it with the constants obtained by direct experiment for that formula; and it is also necessary that all the attending conditions should be the same as those under which the constants were determined. One, therefore, cannot correctly apply the Francis constants when reducing for velocity of approach by the method adopted by Fteley and Stearns, nor *vice versa*. In a similar manner, the Bazin constants are not directly applicable to the formulas obtained by Francis, and by Fteley and Stearns. If our text books had been more clear, there would have been less confusion regarding this point.

If the volume over a standard 16-ft. weir was not correctly obtained as above indicated, the subsequent work based upon these results would need correcting. It is to be hoped that the results as finally determined will be affected by no error which it is possible to eliminate.

Since, as a basis of the experiments performed at Cornell, the whole subject of weir flow is under discussion, it will probably be instructive to give a summary of the results of some of the deductions which the writer has made of the work of Francis, Fteley and Stearns, and Bazin.

As already stated, in order to obtain the most accurate results, it is necessary to duplicate the conditions of some weir for which coefficients have been determined by volumetric measurement, and then to use the same formula and the same coefficients derived from that weir. As a matter of fact, this is seldom possible in practice. The engineer is thus forced to resort to some modified method which, in his judgment, will give results most nearly correct under the circumstances.

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* Since this discussion was received, the statement in the original paper has been changed, and now reads:

n = a coefficient which depends on p and h. (p = height of crest of weir above bottom of channel of approach, and h = observed head on crest.)

Mr. Parmley. The writer has constructed recently about forty weirs of various sorts, and not one of them conforms to any weir for which coefficients have been determined. Most of these weirs are in sewer outlets where the channel of approach is circular or nearly so. In all these weirs, with considerable head, the velocity of approach exceeds that of any upon which experiments have been conducted. While recognizing the desirability of constructing weirs without end contractions, in most cases it has been practically impossible, and where it could have been done, the cost precluded a weir of that form.

A close study, therefore, has been made of the work of these experimenters, in order, if possible, to discover a method which would utilize the results of all, when working with a weir unlike any which had been calibrated. One of the first things noticed was the fact that in correcting for velocity of approach, Bazin followed practically the method of Francis, but with a coefficient which gives a larger correction. There are two methods of making this correction: The theoretical method of Francis, in which the effective head, H', corrected for initial velocity of the water is

$$H' = \left[\left(H + \frac{v^2}{2\,g}\right)^{\frac{3}{2}} - \left(\frac{v^2}{2\,g}\right)^{\frac{3}{2}}\right]^{\frac{2}{3}}$$

and the empirical method used by Fteley and Stearns, in which the effective head is

$$H' = H + c\,\frac{v^2}{2\,g}$$

in which c is some numerical constant to be determined by experiment.

Now, the correction for velocity of approach, to be applied to the coefficient for flow over a weir where there is no velocity of approach, adopted by Bazin, is

$$K = 1 + 0.55\left(\frac{H}{H + P}\right)^2$$

which is identical in form with a formula, expressing the same thing, derived by Hunking and Hart:*

$$K = 1 + 0.2489\left(\frac{H}{H + P}\right)^2$$

The value of K, by Bazin, differs only by the numerical coefficient, and gives a correction 2.2 times that of the formula of Hunking and Hart; but the values given by the latter equation were found to agree within one-two hundredth of 1% of the results obtained by the use of the Francis formula for effective head, given above. It thus appears that the correction for velocity of approach, as deduced by Bazin for use in the formulas for the weirs which he experimented with, is 2.2 times the correction that Francis found necessary to the formula in the reduction of his experiments.

* *Journal of the Franklin Institute*, Vol. 118 (Aug., 1884), p. 121.

The ratio $\frac{H}{H+P}$ was derived from rectangular channels and for Mr. Parmley. suppressed weirs, and is, therefore, the ratio of the sectional area of the water upon the weir to the sectional area of the water in the channel of approach. Adopt the notation $\frac{a}{A}$ to represent this ratio and we may then apply the results of experiments upon suppressed weirs in rectangular channels, to weirs with end contractions in channels having any shape, provided the proper correction for end contraction be made. The formula of Francis, then, for suppressed weirs of such height that there is velocity of approach, becomes

$$Q = 3.33 \left[1 + 0.2489 \left(\frac{a}{A}\right)^2\right] L H^{\frac{3}{2}}$$

and Bazin's formula for the same case becomes

$$Q = C' \left[1 + 0.55 \left(\frac{a}{A}\right)^2\right] L H^{\frac{3}{2}}$$

in which C' = the coefficient where there is no velocity of approach.

In Bazin's formula, as thus modified, C' is not constant, as found to be nearly the case with the limited number of weirs experimented on by Francis, but varies from 3.59 for a head of 0.16 ft. to 3.28 for a head of 1.969 ft., and in the coefficient K, where the corrective term is 2.2 times that of Francis, as before stated.

Outside of the few experiments of Fteley and Stearns, the work of Francis is practically all that we have bearing on the subject of the effect of end contraction. As it was found by Fteley and Stearns to vary from 0.06 to 0.12 of the head, and as their experiments are not extensive enough for us to determine in most cases just what correction to make, the average correction of Francis seems to be the most practicable value to use, viz., 0.10 of the head deducted from the length of the weir for each end contraction. A general formula for sharp-crested weirs, either suppressed or contracted, may then be written

$$Q = C' K [L - 0.1\, n\, H]\, H^{\frac{3}{2}}$$

in which n is the number of end contractions and K is the coefficient obtained by Bazin, since his experiments cover so large a number of conditions.

The values of C', as deduced by Bazin, do not fit exactly the work of American experimenters, and an investigation of this point has been made. The values of C', as deduced from the experiments of Francis, and Fteley and Stearns, after the correction for end contraction and velocity of approach had been made, are plotted in Fig. 29. The values of Bazin, for the case of no end contraction and no velocity of approach, are also plotted. In this manner the variations in the results of the experiments are set clearly before the eye. By the curve it is seen that the values of Bazin agree well with each other,

Mr. Parmley. but that he uniformly gives higher values than do either of the other experimenters. In order to write a formula capable of covering, with a fair degree of accuracy, a wide range of conditions and still give proper weight to the results of all three authorities, values of C' are taken from the curve which is drawn in an intermediate position.

Values of K for variations in the ratio $\frac{a}{A}$ from 0.01 to 0.60 are shown in Table No. 11, together with the values of C' taken from the intermediate curve of Fig. 29.

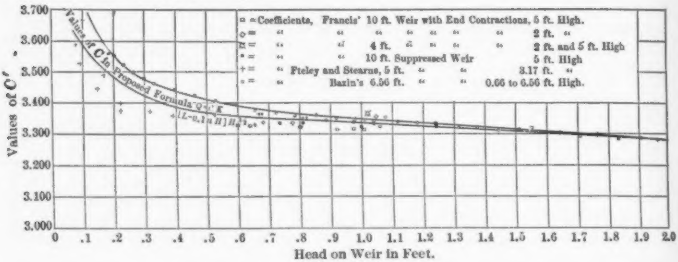


FIG. 29.

The formula, then, with the aid of the auxiliary table for C' and K, is capable of application to sharp-crested weirs, with or without end contractions, with any form of channel of approach and with a great range in the velocity of approach, these corrections being applied separately, and provided for by the coefficients in the general formula.

In order to make a direct comparison of the two methods of reducing for velocity of approach, we may consider any given quantity of flow to be calculated by the two formulas. By equating these expressions for volume we can obtain values of the coefficients that will satisfy the equation. Then let

$$Q = C' \, K \, L \, H^{\frac{3}{2}} = C' \, L \, (H + \, ch)^{\frac{3}{2}}$$

from which

$$K^{\frac{2}{3}} \, H = (H + ch)$$

or,

$$c = \frac{H}{h} (K^{\frac{2}{3}} - 1).$$

But

$$h = 0.01555 \, v^2 = 0.01555 \, \frac{Q^2}{A^2} = \frac{0.01555 \, (C')^2 \, K^2 \, L^2 \, H^3}{A^2}$$

$$= 0.01555 \, (C')^2 \, K^2 \, H \frac{a^2}{A^2}$$

From the equation for K, given above, we have

$$A^2 = \frac{0.55 \, a^2}{K - 1}$$

Substituting this value of A^2 in the equation for h, and then substituting the value of h in the equation for c and reducing gives Mr. Parmley.

$$c = \frac{35.4\,(K^{\frac{2}{3}} - 1)}{(C')^2\, K^2\, (K - 1)}$$

Since, in the second member of the last equation, there are two variables, C' and K, which are given for any depth of weir and any head upon the weir, the equation enables us to determine, after having found the volume by the equation $Q = C'\ K\ L\ H^{\frac{3}{2}}$, precisely what value of c would have been necessary to use in the formula $Q = C'\ L\ (H + ch)^{\frac{3}{2}}$ in order to obtain the same volume.

TABLE No. 11.—Values of the Coefficients in the Formula $Q = C'\ K\ (L - 0.1\ n\ H)H^{\frac{3}{2}}$

$a = (L - 0.1\ n\ H)\ H;$
A = Area of water section in channel of approach;
$\frac{a}{A}$ = Ratio of area of weir section to area of section of channel of approach;
K = Constant for given value of $\frac{a}{A}$;
C' = Constant for given value of H.

$\frac{a}{A}$	K.	$\frac{a}{A}$	K.	H.	C'.
0.01	1.0001	0.31	1.0529	0.10	3.580
0.02	1.0002	0.32	1.0563	0.15	3.520
0.03	1.0005	0.33	1.0599	0.20	3.478
0.04	1.0009	0.34	1.0636	0.25	3.444
0.05	1.0014	0.35	1.0674	0.30	2.420
0.06	1.0020	0.36	1.0713	0.35	3.400
0.07	1.0027	0.37	1.0753	0.40	3.385
0.08	1.0035	0.38	1.0794	0.45	3.376
0.09	1.0044	0.39	1.0837	0.50	3.368
0.10	1.0055	0.40	1.0880	0.55	3.362
0.11	1.0066	0.41	1.0925	0.60	3.358
0.12	1.0079	0.42	1.0970	0.65	3.354
0.13	1.0093	0.43	1.1017	0.70	3.351
0.14	1.0108	0.44	1.1065	0.75	3.349
0.15	1.0124	0.45	1.1114	0.80	3.346
0.16	1.0141	0.46	1.1164	0.85	3.343
0.17	1.0159	0.47	1.1215	0.90	3.340
0.18	1.0178	0.48	1.1267	0.95	3.337
0.19	1.0198	0.49	1.1321	1.00	3.334
0.20	1.0220	0.50	1.1375	1.10	3.329
0.21	1.0243	0.51	1.1431	1.20	3.324
0.22	1.0266	0.52	1.1487	1.30	3.319
0.23	1.0291	0.53	1.1545	1.40	3.313
0.24	1.0317	0.54	1.1604	1.50	3.307
0.25	1.0344	0.55	1.1664	1.60	3.301
0.26	1.0372	0.56	1.1725	1.70	3.296
0.27	1.0401	0.57	1.1787	1.80	3.290
0.28	1.0431	0.58	1.1850	1.90	3.285
0.29	1.0463	0.59	1.1915	2.00	3.280
0.30	1.0495	0.60	1.1980	....	.....

Mr. Parmley. Fig. 30 shows the value of c for several weirs and heads as given by Fteley and Stearns* and the values for the same volumes flowing when calculated by the formula $Q = C' K L H^{\frac{3}{2}}$. The values by Fteley and Stearns are shown by broken lines, and those by the latter formula by solid lines.

It will be seen that the values of c by the two methods for a weir 1 ft. high are almost identical, but that for higher weirs, with consequently lower velocities of approach, the values by Fteley and Stearns are lower. On the other hand, by the Bazin formula with modified coefficients, the values of c increase as the weirs become higher and the velocities of approach become less. It would seem that the latter condition is

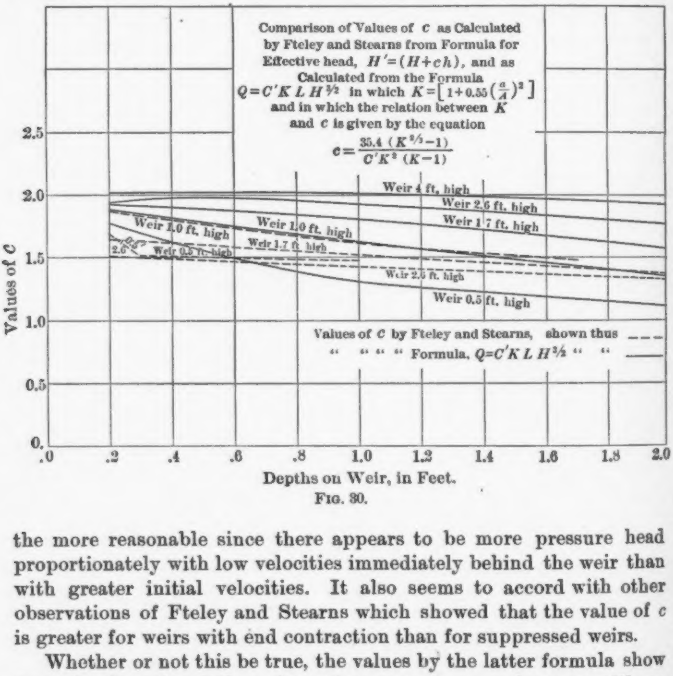


FIG. 30.

the more reasonable since there appears to be more pressure head proportionately with low velocities immediately behind the weir than with greater initial velocities. It also seems to accord with other observations of Fteley and Stearns which showed that the value of c is greater for weirs with end contraction than for suppressed weirs.

Whether or not this be true, the values by the latter formula show a regular law of variation, since c increases in every instance with an increase in the height of the weir, while the values as given by Fteley and Stearns for a weir 1.7 ft. high are intermediate between those for weirs 1 ft. and 0.5 ft. high; and for a weir 2.6 ft. high they are less than those for any of the others. Such results are anomalous, and

* *Transactions*, Am. Soc. C. E., Vol. xii, Table No. III, p. 22.

Mr. Parmley.

seem to show that the values derived by their experiments were affected by some peculiar condition which may not occur in other cases. In Table No. 12 a direct comparison of the values of c is shown for the cases having one or more end contractions, as given by Fteley and Stearns.* The values of c are calculated by the formula for c, given above. From this, together with the data in the table of Fteley and Stearns, a comparison of the velocity heads is made, with the difference between them shown in the last column.

TABLE No. 12.—COMPARATIVE VALUES OF c IN THE FORMULA FOR EFFECT OF VELOCITY OF APPROACH $H' = (H + c\,h)$ AS GIVEN BY FTELEY AND STEARNS, AND AS GIVEN BY THE FORMULA $c = \dfrac{35.4\,(K^{\frac{3}{2}} - 1)}{(C')^2\,K^2\,(K - 1)}$.

No. of experiment.	Height of weir.	Length of weir.	Number of end contractions.	Measured head on weir.	Value of c given by Fteley and Stearns.	Value of c given by formula.	Value of ch given by Fteley and Stearns.	Value of h by Fteley and Stearns.	Value of h by formula for c.	Column (10) deducted from column (9).
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	3.56	3.0	2	0.8702	†2.14	2.08	0.0042	0.0020	0.0020	0.0000
2	1.70	3.0	2	0.8612	2.26	2.03	0.0132	0.0058	0.0065	— 0.0007
3	1.00	3.0	2	0.8490	2.27	1.95	0.0254	0.0112	0.0130	— 0.0018
4	0.50	3.0	2	0.8310	2.00	1.83	0.0434	0.0217	0.0237	— 0.0020
5	3.56	3.3	1	0.5765	†2.02	2.08	0.0017	0.0008	0.0008	0.0000
6	1.00	3.3	1	0.5647	2.35	1.96	0.0135	0.0057	0.0069	— 0.0012
7	0.50	3.3	1	0.5574	1.65	1.83	0.0208	0.0126	0.0114	+ 0.0012
8	3.56	3.3	1	0.8062	†2.00	2.03	0.0040	0.0020	0.0020	0.0000
9	2.60	3.3	1	0.8036	1.99	2.05	0.0066	0.0033	0.0032	+ 0.0001
10	1.70	3.3	1	0.7969	2.16	2.01	0.0133	0.0062	0.0066	— 0.0004
11	1.00	3.3	1	0.7850	2.09	1.92	0.0252	0.0121	0.0131	— 0.0010
12	0.50	3.3	1	0.7677	1.78	1.77	0.0425	0.0239	0.0240	— 0.0001
13	3.56	3.3	1	0.9307	†1.99	2.09	0.0058	0.0029	0.0028	+ 0.0001
14	1.00	3.3	1	0.9024	2.10	1.90	0.0341	0.0162	0.0179	— 0.0017
15	0.50	3.3	1	0.8775	1.91	1.75	0.0590	0.0309	0.0337	— 0.0028
16	3.56	4.0	1	0.7048	†1.81	2.05	0.0038	0.0021	0.0019	+ 0.0002
17	0.50	4.0	1	0.6639	1.58	1.67	0.0447	0.0283	0.0268	+ 0.0015

The calculations are based on the weirs shown in *Transactions*, Am. Soc. C. E., Vol. xii, Table No. III, pp. 22 and 23.

By the last column of differences it will be observed that the values as calculated are within the limits of error in making the observation of the head upon the weir, and this fact, together with the probability that some small errors in readings were made by Fteley and Stearns, seems to make the calculated value of c practically exact in any given case.

To test the preceding work still further, and if possible remove any doubt there may be in its reliability, the volumes of discharge

* *Transactions*, Am. Soc. C. E., Vol. xii, Table No. III, pp. 22 and 23.

† Value used to determine effective depth on weir, and not the result of experiment.

Mr. Parmley. have been calculated for the various experiments made by Francis, and Fteley and Stearns, so that these calculated volumes could be compared with the volumes obtained by them from direct volumetric measurement. These results, together with the percentage of variation of calculated from measured volumes, are shown in Tables Nos. 13 to 17.

TABLE No. 13.—Experiments by Francis.

Case I.—Length of weir, 9.997 ft.; width of channel above weir, 13.96 ft.; mean depth of channel below crest of weir, 6 ft., above weir, 5.048 ft.

No. of Francis' Experiments (Mean of Nos.).	Measured head.	Actual Q, in cubic feet.	Q' by formula, in cubic feet.	$\frac{Q'}{Q}$
72, 73, 74	0.5983	15.2466	15.397	1.010
76, 77, 78	0.63823	16.7777	16.942	1.010
75	0.65525	17.4305	17.612	1.010
56, 57, 58, 59, 60, 61	0.79899	23.4304	23.66	1.009
22, 23	0.9218	28.9334	29.20	1.009
11, 17, 18, 19, 24, 31, 32, 33	0.9701	31.2479	31.466	1.007
12, 20, 21, 25, 26, 27	1.0050	32.9542	33.16	1.006
13, 14, 15, 16, 28, 29	1.0390	34.5975	34.823	1.007
30	1.0692	36.1752	36.313	1.004
5, 6, 7, 8, 9, 10	1.2476	45.5654	45.60	1.001
1, 2, 3, 4	1.5508	62.602	62.60	1.000

Case II.—Same conditions as in Case I, except that weir was divided into two bays by a central partition 2 ft. wide. Each resulting weir, 3.999 ft. long.

34, 35	1.0182	13.138	13.06	0.994

Case III.—Weir 9.997 ft. long; complete end contraction; depth of channel below crest of weir, 2.014 ft.; width of channel, 13.96 ft.

79, 80, 81, 82, 83, 84	0.6493	17.3400	17.60	1.015
62, 63, 64	0.7876	23.2658	23.518	1.011
65, 66	0.8841	27.7038	27.704	1.010
36, 37, 38, 39, 40, 41	1.0420	35.5792	35.820	1.007
42, 43	1.0753	37.2693	37.55	1.007

Case IV.—Same conditions as in Case III, except that the weir was divided into two bays by a central partition 2 ft. wide, thus making two weirs each 3.999 ft. long.

85, 86, 87, 88	0.6800	7.2739	7.352	1.011

Case V.—Suppressed weir, 9.995 ft. long; width of channel, 9.992 ft.; depth below crest of weir, 5.048 ft.

67	0.7362	21.1532	21.3[illegible]	1.009
68, 69, 70, 71	0.8099	24.4498	24.629	1.007
44, 45, 46, 47, 48, 49, 50	0.9790	32.5616	32.751	1.006
51, 52, 53, 54, 55	1.00026	33.4946	33.50	1.010

TABLE No. 14.—EXPERIMENTS BY FTELEY AND STEARNS. Mr. Parmley.

Sharp-crested weir with end contractions suppressed; weir 3.17 ft. high above bottom of channel; rectangular channel of approach.

No. of Experiment.	Length of weir.	Measured head.	Actual Q, in cubic feet.	Q' by formula, in cubic feet.	$\frac{Q'}{Q}$
1	5.000	0.8198	12.750	12.66	0.9944
3	5.000	0.8118	12.466	12.519	1.004
4	5.000	0.6761	9.430	9.473	1.005
5	4.997	0.6713	9.322	9.379	1.006
6	4.996	0.5203	9.342	6.385	1.007
8	4.999	0.4761	5.547	5.590	1.008
9	4.999	0.4569	5.199	5.257	1.011
10	4.999	0.3890	4.094	4.138	1.011
12	4.994	0.3407	3.3540	3.402	1.014
13	4.998	0.3114	2.9355	2.980	1.015
14	4.999	0.2598	2.2415	2.284	1.019
15	4.999	0.2467	2.0780	2.115	1.018
17	5.000	0.2190	1.7474	1.779	1.018
18	5.000	0.2182	1.7211	1.769	1.028
19	5.000	0.2476	1.7837	1.762	0.988
20	4.998	0.1650	1.1705	1.177	1.005
21	4.995	0.1627	1.1367	1.153	1.014
22	4.995	0.1444	0.9469	0.9681	1.022
23	4.998	0.1235	0.7495	0.7452	0.994
24	4.998	0.1225	0.7526	0.7623	1.013
27	4.996	0.1009	0.5846	0.5731	0.978
28	4.996	0.1008	0.5877		
29	4.996	0.0991	0.5498	0.5582	1.015
31	4.996	0.0746	0.3652	0.3648	0.999

TABLE No. 15.

Sharp-crested weir with end contractions suppressed; weir 18.996 ft. long and 6.55 ft. high above bottom of channel.

Number of Experiment.	Measured head.	Actual Q, in cubic feet.	Q' by formula, in cubic feet.	$\frac{Q'}{Q}$
1	1.6038	130.117	130.341	1.002
2	1.4546	112.066	112.313	1.002
3	1.2981	94.192	94.652	1.005
5	1.1456	77.783	78.415	1.008
6	0.9873	62.061	62.713	1.011
7	0.9864	62.023	62.609	1.010
8	0.8191	46.760	47.491	1.016
9	0.6460	32.685	33.226	1.017
10	0.4685	20.178	20.596	1.021

TABLE No. 16.

Experiments on weirs in channel 5 ft. wide, and with height of weirs 3.56 ft. above bottom of channel; dimensions of weirs as shown in the table:

Number designating form of weir.	Number of end contractions.	Length of weir.	DISTANCE FROM SIDE OF CHANNEL TO THE END OF WEIR.	
			North side.	South side.
1	0	5.0	0.0	0.0
2	1	4.0	1.0	0.0
3	1	4.0	0.0	1.0
4	1	3.3	0.0	1.7
5	2	3.0	1.0	1.0
6	2	2.3	1.0	1.7

Mr. Parmley.

TABLE No. 17.

Number of Experiment.	Number designating form of weir by Table No. 16.	Measured head, in feet.	Actual Q, in cubic feet.	Q' by Formula, in cubic feet.	$\frac{Q'}{Q}$
1	1	0.1509	1.007	1.034	1.026
4	5	0.2155	1.007	1.028	1.021
6	2	0.2303	1.868	1.917	1.026
7	2	0.2691	1.868	1.911	1.023
8	3	0.2693	1.869	1.913	1.024
9	5	0.3301	1.869	1.905	1.019
10	1	0.2304	1.869	1.918	1.026
11	1	0.3368	3.283	3.343	1.018
12	2	0.3942	3.283	3.336	1.016
13	3	0.3944	3.283	3.340	1.017
14	5	0.4843	3.284	3.317	1.010
15	4	0.4498	3.284	3.337	1.016
16	6	0.5824	3.284	3.287	1.001
17	1	0.3369	3.285	3.380	1.019
18	1	0.4244	4.636	4.707	1.015
19	3	0.4978	4.636	4.705	1.015
20	4	0.5678	4.637	4.697	1.013
21	1	0.4245	4.637	4.719	1.015
22	1	0.4308	4.741	4.813	1.015
23	5	0.6215	4.736	4.761	1.005
24	6	0.7478	4.736	4.701	0.992
25	4	0.5764	4.736	4.805	1.015
26	4	0.5766	4.736	4.807	1.015
27	1	0.4302	4.731	4.814	1.015
28	1	0.5115	6.132	6.217	1.014
29	2	0.5997	6.134	6.197	1.010
30	3	0.5996	6.134	6.196	1.010
31	5	0.7398	6.134	6.130	1.000
32	4	0.6860	6.134	6.212	1.013
33	6	0.8905	6.134	6.019	0.981
34	1	0.5117	6.136	6.221	1.014
35	6	0.9548	6.796	6.640	0.977
36	1	0.5477	6.796	6.907	1.016
37	1	0.6010	7.814	7.918	1.013
38	2	0.7055	7.815	7.892	1.010
39	3	0.7064	7.815	7.905	1.011
40	5	0.8714	7.815	7.769	0.994
41	1	0.6011	7.816	7.920	1.013
42	4	0.8062	7.815	7.889	1.009
43	4	0.8063	7.804	7.886	1.010
44	1	0.6002	7.798	7.902	1.013
45	5	0.8702	7.800	7.752	0.994
Mean of 46, 47	1	0.69245	9.677	9.805	1.013
48	4	0.9317	9.667	9.755	1.009
49	4	0.9297	9.630	9.724	1.010
50	1	0.6897	9.618	9.743	1.013
51	2	0.9448	12.147	12.174	1.002
52	3	0.9432	12.147	12.137	1.001
53	1	0.8047	12.147	12.311	1.013
54	2	0.9460	12.147	12.205	1.004

It is thus seen that the formula, when applied to the experiments of Francis, and of Fteley and Stearns, agrees within about 1%, usually, even through a very wide range of conditions. For the most part, the formula gives results too large for these experiments. If applied to the work of Bazin, on the other hand, the results for small heads will be found to be too small by about the same percentage, and for larger heads the error approaches zero. The formula, with the coefficients as they stand, gives about equal weight to the work of all these

experimenters, and may therefore be used with a good deal of confidence in cases where the conditions are not exactly like those of weirs actually tested. In general, it will be noticed that the percentage of variation is greater for small depths than for large depths on the weir. As, for small heads, the experimental errors are also proportionately greater, the errors of the formula are of less consequence than they would be if they increased with the depth on the weir. Mr. Parmley.

A serious difficulty with the method of calculating the effective head by the formula $H' = H + c\,h$ is the fact that the value of c varies, not only with the varying head, but with every depth of weir, and, as Fteley and Stearns found, with the condition as to end contraction. This difficulty is in part obviated by the foregoing process.

To the writer's mind, the conclusion to be drawn from the foregoing discussion is that the methods adopted by Francis and by Bazin when investigating the effect of velocity of approach are preferable to the method used by Fteley and Stearns. This conclusion seems to be confirmed by the fact that it is possible to write a single formula, with a supplemental table of values of C', depending on the head upon the weir, and a table of values of K, depending on the relative areas of water section on the weir and in the channel of approach, which will give results nearly approximating the true quantities of discharge for a very wide range of conditions.

It is to be hoped that the experiments, so well begun at Cornell University, may be continued until the subject of weir flow is taken out of the cloud of uncertainty in which at present it is enveloped.

George T. Nelles, M. Am. Soc. C. E. (by letter).—Only those engineers, whose practice has shown them the difficulties and uncertainties involved in the determination of the flow of water over dams of practical form and dimensions, will ever fully realize the great service rendered to the profession by the experiments of Bazin and those made for the U. S. Board of Engineers on Deep Waterways by the author, at the Cornell Hydraulic Laboratory. Mr. Nelles.

The Cornell experiments are presented in a practical and useful form and admit of an easy and direct application to the solution of questions concerning the flow of water over dams, met with in everyday practice by the working engineer.

Early in the course of a recent investigation to determine the probable effect of the construction of several large dams in the Tennessee River, for the purpose of producing slack-water navigation, the writer recognized the inapplicability of the existing published data, based on small-scale experiments, and made a systematic effort to secure information of a more practical nature, founded on actual observations on the flow over existing dams. His efforts in this direction, which comprised a study of the available literature on the subject, and correspondence with many of the most eminent hydraulic

Mr. Nelles. engineers of this country and Europe, met with small success, and exhibited the paucity of exact knowledge of this nature.

The best results obtained, relating to the free discharge into air, were as follows:

(1) From Dwight Porter, M. Am. Soc. C. E., five observations, made in 1894 by two students of the Massachusetts Institute of Technology, on the flow over a dam in the Blackstone River, at Albion, Mass. The dam is 217 ft. long, with a longitudinal timber crest 12 ins. wide; the lower face is vertical, and the upper face is covered with rip-rap stone, sloping at about 30° with the horizontal; the height of the dam is about 8 ft. The discharge was measured by meter at a section 500 ft. below the dam. The head was measured by a hook gauge, located 20 ft. above the crest of the dam.

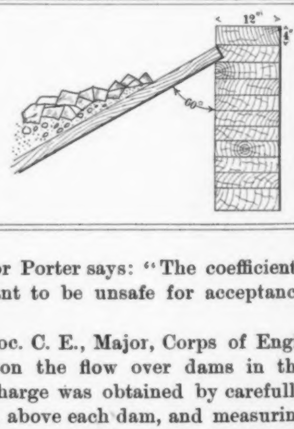
These observations when applied to the Bazin formula

$$Q = m\ L\ h \sqrt{2\,g\,h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a),$$

in which m represents the coefficient of discharge, L the crest length of the dam and h the observed head, give the values for the coefficient m shown in Table No. 18.

TABLE No. 18.

Observation.	Observed head, in feet.	Coefficient m.
(1)............	0.735	0.439
(2)............	0.867	0.441
(3)............	0.829	0.456
(4)............	0.973	0.491
(5)............	1.025	0.470



Concerning these results, Professor Porter says: "The coefficients are sufficiently high and discordant to be unsafe for acceptance unless otherwise confirmed."

(2) From W. H. Bixby, M. Am. Soc. C. E., Major, Corps of Engineers, U. S. A., four observations on the flow over dams in the Muskingum River, Ohio. The discharge was obtained by carefully sounding the area of a section 500 ft. above each dam, and measuring the velocity by long rod floats. The mean velocity was deduced from the observed, by the formula given in Johnson's "Surveying."

The dams are constructed of timber cribs, filled with stone, and have planked sloping crests. The upper face is 10 ft. long, with a slope of 1 ft. The lower face is 30 ft. long, with a slope of 4 ft.*

* *The Engineering Record*, March 8th, 1890, page 216.

Table No 19 gives the results of these observations. Mr. Nelles.

TABLE No. 19.

Number of dam.	Length on crest, in feet.	Mean height, in feet.	Area of discharge section, in square feet.	Discharge, in cubic feet per second.	Mean velocity, in feet per second.	Fall over dam, in feet.	Observed depth on crest, in feet.	Coefficient, in the Bazin formula (m).
3	848	12.6	7 765	18 118	2.333	8.00	2.86	0.551
4	535	15.9	8 360	25 559	3.045	6.70	4.66	0.589
7	472	14.2	8 230	21 015	2.553	7.00	4.40	0.600
8	515	16.0	7 330	22 310	3.044	5.16	5.90	0.376

T. C. Clarke, M. Am. Soc. C. E., gives certain results* concerning the flow over a dam in the Ottawa River, from which the following is abstracted. The dam is 30 ft. high, built of stone-filled timber cribs, with upper and lower faces planked and sloping at 3 horizontal to 1 vertical. The data are shown in Table No. 20.

TABLE No. 20.

Length, in feet.	Depth on crest, in feet.	Discharge, in cubic feet per second.	Coefficient in the Bazin formula (m).
1 600	2.5	26 000	0.512
1 760	10.0	190 000	0.425

In a paper read before the American Society of Mechanical Engineers in 1897, Professor R. C. Carpenter, of Cornell University, gave the results of a number of observations made by him at the Bridgeport Pumping Station, Chicago, on the flow over a thin vertical weir, 30 ft. long and 8.7 ft. high, with a sharp crest.

The discharge was measured by carefully conducted rod float observations, and checked by the known capacity of the basin. After correcting for two end contractions, Professor Carpenter gives the figures shown in Table No. 21.

TABLE No. 21.

Observed heads, in feet, corrected for velocity of approach	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Coefficient C in Francis' formula	3.657	3.609	3.577	3.560	3.545	3.537	3.529

The coefficients given by the Massachusetts Institute of Technology observations and the Muskingum River observations have been plotted on the diagram, Fig. 31, in comparison with Bazin's Series No. 178; and appear to justify the early conclusions drawn from the Bazin experiments, that for weirs with sloping faces, the coefficients increase

* *Transactions*, Am. Soc. C. E., Vol. xxxiv, page 508.

Mr. Nelles. with the head. When, however, we compare these coefficients with the Cornell experiments, of which Series No. 7 is taken as a type, and shown in Fig. 31, we are met by complications which give rise to just doubts as to the accuracy of the observations in question, or the applicability of experiments, even on the large scale of those carried out at Cornell, to the flow over structures differing in shape and dimensions from the experimental forms. While both causes may combine to produce the apparent discrepancies, the former is doubtless the leading factor. The condition of the nappe, if known, would probably account, in some degree, for the discrepancies noted.

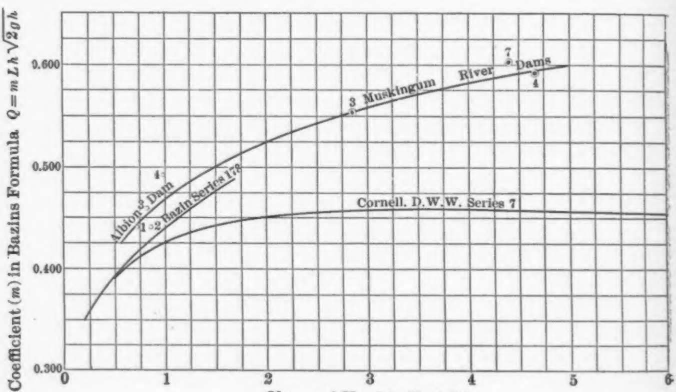


FIG. 31.

The heads corresponding to the curve representing the Cornell D. W. W. Series No. 7 have been corrected for velocity of approach.

FLOW OVER SUBMERGED DAMS.

The practical solution of most questions concerning the free flow over weirs and dams of the usual forms is rendered comparatively easy and simple by the experiments recorded in this paper. The solution of questions concerning the flow over submerged or drowned weirs is still, however, involved in doubt and uncertainty, although the recent experiments by Bazin, covering this form of flow, have added greatly to our knowledge, and afford a basis for a comprehensive study, which will doubtless eventually lead to a better, if not a complete, understanding of the governing principles. A knowledge of this form of flow, which is of relatively less importance to engineers in general than a knowledge of free flow, is nevertheless essential in the study of various engineering problems.

Mr. Nelles.

The experiments of Francis, Fteley and Stearns, Bazin, and others, have determined, within the limits of their experiments, the law of variation of the coefficient of discharge for submerged flow over thin sharp-crested vertical weirs. The results of and the deductions from the experiments of Francis, and Fteley and Stearns, are to be found in the *Transactions* of this Society,* to which reference is made. Bazin has applied the results of his experiments on submerged flow over weirs with sharp crests, in thin partition, to the same formula he used for free discharge into air,

$$Q = m\,L\,h\,\sqrt{2\,g\,h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

and has deduced the two expressions (Equations 11 and 12, page 265), for the coefficient of discharge m. In these equations (11 and 12) m' represents the coefficient of free discharge for a similar weir of the same height, with the same depth h of water in the pool above the weir on the crest. The depth of the water in the pool below the weir on the crest is represented by h_1; p represents the height of the weir, and $z = h - h_1 =$ the difference in level of the water surface above and below the weir. These expressions, in connection with the well-determined coefficients of free discharge given in Table No. 2 afford an easy and safe means of obtaining the coefficient for submerged flow for the conditions and limits comprised in Bazin's experiments, Series Nos. 64 to 85, 4th article, 1894.

In his 5th article, 1896, Bazin treats of the submerged flow over vertical weirs with flat crests (*deversoirs à poutrelles*), made up of square timbers of the same width as the crest. Concerning these experiments he says:

"Independent of the experiments we have just studied (experiments on the free flow over similar weirs), there have been made, by raising the level of the water surface below the weirs, four other series, on weirs 0.75 m. (2.46 ft.) and 0.35 m. (1.15 ft.) high above the bed of the channel, with crests 0.20 m. (0.66 ft.) and 0.10 m. (0.33 ft.) wide.

"They are entirely analogous to those which were made on vertical weirs in thin partition† (Series 64 to 85), and exactly the same methods have been followed in measuring the discharge, pressure, etc.

"The discharge was regulated to correspond to constant heads of 0.10 m, 0.15 m. * * * * 0.40 m., 0.45 m (0.325 ft, 0.492 ft. * * * * 1.148 ft., 1.312 ft.), on the standard weir of comparison. When the discharge corresponding to a given head became constant, then the level of the water surface below the weir was successively changed to $0.04p$, $0.08p$, $0.12p$, etc. ($p =$ height of weir), both above and below the crest, as was done in the similar experiments on weirs in thin partition. Finally, a similar series was executed on a vertical weir 2.0 m. (6.56 ft.) wide."

* *Transactions*, Am. Soc. C. E., Volumes xii, xiii and xiv.

† In his articles Bazin refers to weirs similar in form to his standard weir, as weirs in thin partition (*Déversoirs en mince paroi*). These weirs are made up of horizontal timbers about 4 ins. square, the crest being formed by an iron plate ¼ in. thick, in the prolongation of the back of the weir and extending 4 ins. above the timbers.

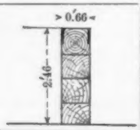
Weirs of other form, with sharp crest, Series 195, 196 and 197, he designates as weirs with sharp crest (*Déversoirs à vive arête*).

Mr. Nelles.

BAZIN'S EXPERIMENTS.—SERIES NOS. 120 TO 123.

Nappe wetted underneath, with water retained in pool below weir. Description of weirs under observation: Height, 0.75 m. = 2.46 ft. and 0.35 m. = 1.15 ft. Crests, 0.20 m. = 0.66 ft. and 0.10 m. = 0.33 ft. wide. Length, 2.0 m. = 6.56 ft.

Dimensions of standard weir used for measuring the discharge: Height, 1.135 m. = 3.72 ft. Length, 2.0 m. = 6.56 ft.

(1)	(2)	(3)	(4)	(5)
	OBSERVED HEADS, IN FEET.		Height of the retained water with reference to crest: + = above, − = below. (h_1)	COEFFICIENTS OF DISCHARGE.
	On weir under consideration. (h.)	On standard weir of comparison. (H.)		For the Bazin Form a ($Q=m\,L\,h\sqrt{2gh}$)
Bazin's Series No. 120.				
1	0.388	0.329	−0.199	0.3379
2	0.394	0.335	−0.097	0.3391
3	0.395	0.336	−0.002	0.3388
4	0.392	0.332	+0.191	0.3373
5	0.402	0.331	+0.294	0.3241
6	0.447	0.331	+0.392	0.2758
7	0.539	0.333	+0.494	0.2100
1	0.548	0.496	−0.392	0.3684
2	0.550	0.496	−0.200	0.3671
3	0.549	0.489	+0.000	0.3602
4	0.558	0.498	+0.098	0.3605
5	0.552	0.484	+0.187	0.3525
6	0.569	0.498	+0.296	0.3502
7	0.569	0.486	+0.392	0.3378
8	0.605	0.498	+0.490	0.3194
9	0.680	0.492	+0.589	0.2629
10	0.760	0.498	+0.686	0.2269
1	0.699	0.657	−0.394	0.3878
2	0.701	0.657	−0.197	0.3858
3	0.710	0.660	−0.001	0.3810
4	0.717	0.662	+0.199	0.3774
5	0.723	0.664	+0.290	0.3737
6	0.723	0.664	+0.295	0.3740
7	0.722	0.655	+0.388	0.3683
8	0.742	0.663	+0.492	0.3589
9	0.765	0.654	+0.593	0.3369
10	0.837	0.660	+0.687	0.2980
11	0.909	0.658	+0.783	0.2621
1	0.842	0.826	−0.589	0.4126
2	0.841	0.818	−0.395	0.4077
3	0.846	0.823	−0.198	0.4081
4	0.852	0.825	−0.004	0.4053
5	0.854	0.824	+0.100	0.4034
6	0.861	0.829	+0.189	0.4021
7	0.862	0.828	+0.193	0.4009
8	0.867	0.830	+0.290	0.3981
9	0.861	0.819	+0.291	0.3952
10	0.861	0.812	+0.419	0.3905
11	0.890	0.820	+0.587	0.3762
12	1.006	0.829	+0.786	0.3184
13	1.131	0.819	+0.980	0.2621
1	0.979	0.998	−0.788	0.4386
2	0.974	0.983	−0.396	0.4318
3	0.987	0.995	−0.199	0.4316
4	0.984	0.985	0.000	0.4266
5	0.990	0.982	+0.197	0.4207
6	1.000	0.989	+0.294	0.4191
7	1.003	0.982	+0.392	0.4130
8	1.005	0.988	+0.393	0.4151
9	1.006	0.979	+0.491	0.4088
10	1.014	0.991	+0.492	0.4112
11	1.020	0.984	+0.588	0.4034
12	1.024	0.987	+0.595	0.4034
13	1.046	0.988	+0.658	0.3917
14	1.102	0.987	+0.788	0.3617
15	1.138	0.988	+0.887	0.3318
16	1.226	0.994	+0.980	0.3111
17	1.288	0.988	+1.082	0.2862
18	1.358	0.992	+1.182	0.2657
1	1.111	1.157	−0.584	0.4541
2	1.117	1.158	−0.397	0.4507
3	1.118	1.153	−0.193	0.4467
4	1.117	1.149	0.000	0.4473
5	1.127	1.149	+0.196	0.4414
6	1.130	1.148	+0.293	0.4378
7	1.142	1.157	+0.390	0.4346
8	1.146	1.150	+0.487	0.4294
9	1.142	1.147	+0.493	0.4304
10	1.156	1.152	+0.588	0.4251
11	1.158	1.150	+0.593	0.4227
12	1.172	1.151	+0.688	0.4158
13	1.174	1.154	+0.690	0.4158
14	1.211	1.150	+0.788	0.3955
15	1.279	1.153	+0.881	0.3653

Mr. Nelles.

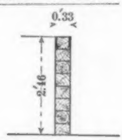
(1)	(2) h	(3) H	(4) h_1	(5) m
16	1.335	1.151	+0.984	0.3420
1	1.243	1.312	—0.394	0.4644
2	1.252	1.317	—0.198	0.4620
3	1.253	1.316	—0.002	0.4615
4	1.259	1.313	+0.195	0.4561
5	1.269	1.308	+0.397	0.4481
6	1.274	1.308	+0.490	0.4451
7	1.288	1.312	+0.590	0.4405
8	1.304	1.317	+0.687	0.4322
9	1.306	1.315	+0.689	0.4201
10	1.326	1.314	+0.784	0.4345
11	1.332	1.315	+0.785	0.4220
12	1.397	1.314	+0.889	0.3907
13	1.446	1.320	+0.985	0.3734
1	1.376	1.479	—0.390	0.4787
2	1.382	1.478	—0.200	0.4753
3	1.396	1.491	+0.001	0.4746
4	1.395	1.479	+0.197	0.4688
5	1.409	1.474	+0.396	0.4635
6	1.423	1.479	+0.589	0.4555
7	1.461	1.480	+0.780	0.4381

Bazin's Series
No. 121.

1	0.386	0.328	—0.193	0.3325
2	0.387	0.329	—0.096	0.3396
3	0.388	0.329	+0.041	0.3383
4	0.388	0.330	+0.086	0.3398
5	0.389	0.331	+0.183	0.3391
6	0.395	0.329	+0.271	0.3295
7	0.423	0.328	+0.371	0.2963
8	0.496	0.325	+0.460	0.2288
9	0.581	0.333	+0.564	0.1883
10	0.667	0.331	+0.655	0.1516
1	0.540	0.495	—0.462	0.3760
2	0.542	0.492	—0.374	0.3714
3	0.546	0.493	—0.187	0.3681
4	0.547	0.492	—0.001	0.3665
5	0.550	0.493	+0.087	0 3632
6	0.550	0.495	+0.184	0.3653
7	0.555	0.494	+0.273	0.3604
8	0.562	0.494	+0.369	0.3532
9	0.579	0.494	+0.455	0.3383
10	0.624	0.493	+0.557	0.2999
11	0.709	0.494	+0.647	0.2492
12	0.780	0.494	+0.735	0.2158
	0.689	0.660	—0.175	0.3987
	0.690	0.661	—0.001	0.3993
	0.699	0.661	+0.184	0.3917
4	0.701	0.659	+0.281	0.3878
5	0.711	0.660	+0.370	0.3809
6	0.716	0.659	+0.459	0.3758
7	0.732	0.658	+0.561	0.3635
8	0.768	0.661	+0.649	0.3405
9	0.841	0.661	+0.742	0.2969
10	0.911	0.657	+0.833	0.2605
11	0.988	0.658	+0.929	0.2308
1	0.829	0.824	+0.001	0.4219
2	0.839	0.825	+0.181	0.4147
3	0.839	0.822	+0.281	0.4127
4	0.847	0.829	+0.368	0.4111
5	0.852	0.828	+0.458	0.4076
6	0.863	0.827	+0.557	0.3992
7	0.871	0.828	+0.618	0.3934
8	0.875	0.819	+0.654	0.3652

(1)	(2) h	(3) H	(4) h_1	(5) m
9	0.906	0.825	+0.732	0.3700
10	0.975	0.828	+0.832	0.3322
11	1.040	0.823	+0.923	0.2994
12	1.112	0.823	+1.020	0.2710
13	1.189	0.824	+1.112	0.2455
1	0.961	0.986	+0.178	0.4432
2	0.968	0.991	+0.281	0.4400
3	0.967	0.986	+0.375	0.4377
4	0.979	0.987	+0.463	0.4315
5	0.986	0.987	+0.558	0.4269
6	1.001	0.988	+0.652	0.4173
7	1.020	0.989	+0.744	0.4067
8	1.019	0.989	+0.745	9.4073
9	1.047	0.985	+0.829	0.3894
10	1.113	0.992	+0.923	0.3583
11	1.177	0.989	+1.017	0.3289
12	1.248	0.989	+1.113	0.3007
13	1.314	0.984	+1.205	0.2760
1	1.093	1.147	+0.281	0.4597
2	1.100	1.150	+0.368	0.4569
3	1.105	1.151	+0.464	0.4544
4	1.115	1.152	+0.557	0.4489
5	1.121	1.150	+0.654	0.4439
6	1.134	1.151	+0.736	0.4368
7	1.158	1.152	+0.828	0.4235
8	1.168	1.147	+0.876	0.4154
9	1.190	1.150	+0.925	0.4054
10	1.255	1.150	+1.018	0.3751
11	1.315	1.151	+1.113	0.3493
12	1.381	1.151	+1.204	0.3249
13	1.439	1.149	+1.293	0.3051
1	1.230	1.317	+0.462	0.4742
2	1.237	1.316	+0.557	0.4693
3	1.242	1.314	+0.653	0.4654
4	1.257	1.315	+0.743	0.4586
5	1.268	1.309	+0.834	0.4484
6	1.294	1.311	+0.923	0.4358
7	1.344	1.311	+1.020	0.4126
8	1.371	1.316	+1.063	0.4024
9	1.398	1.312	+1.115	0.3890
10	1.453	1.314	+1.204	0.3684
11	1.509	1.315	+1.294	0.3479
1	1.336	1.478	+0.659	0.4995
2	1.376	1.476	+0.747	0.4763
3	1.395	1.485	+0.829	0.4721
4	1.405	1.477	+0.922	0.4628
5	1.441	1.476	+1.024	0.4447
6	1.495	1.478	+1.114	0.4222
7	1.542	1.478	+1.203	0.4029
8	1.587	1.477	+1.292	0.3861

Bazin's Series
No. 122.

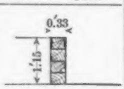


1	0.360	0.330	—0.187	0.3792
2	0.353	0.322	—0.098	0.3780
3	0.361	9.328	0.000	0.3755
4	0.371	0.330	+0.095	0.3639
5	0.372	0.327	+0.192	0.3571
6	0.376	0.337	+0.294	0.3681
1	0.495	0.502	—0.381	0.4354
2	0.495	0.489	—0.198	0.4187
3	0.504	0.494	—0.002	0.4155
4	0.515	0.495	+0.198	0 4036

Mr. Nelles.

(1)	(2) h	(3) H	(4) h_1	(5) m
5	0.523	0.493	+0.293	0.3923
6	0.579	0.492	+0.390	0.3352
7	0.756	0.494	+0.488	0.2259
1	0.631	0.658	—0.401	0.4529
2	0.634	0.658	—0.199	0.4505
3	0.640	0.657	0.000	0.4476
4	0.644	0.650	+0.197	0.4324
5	0.654	0.654	+0.292	0.4251
6	0.718	0.655	+0.396	0.3713
7	0.808	0.658	+0.596	0.3130
1	0.751	0.813	—0.590	0.4805
2	0.768	0.824	—0.393	0.4734
3	0.766	0.821	—0.199	0.4733
4	0.771	0.819	+0.002	0.4667
5	0.779	0.813	+0.196	0.4543
6	0.841	0.823	+0.299	0.4126
7	0.853	0.812	+0.387	0.3959
8	0.887	0.817	+0.489	0.3758
8	0.931	0.823	+0.592	0.3541
10	1.016	0.815	+0.784	0.3059
1	0.880	0.987	—0.791	0.5064
2	0.879	0.980	—0.589	0.5117
3	0.884	0.983	—0.411	0.5001
4	0.890	0.991	—0.398	0.5010
5	0.889	0.982	—0.199	0.4946
6	0.897	0.978	—0.001	0.4859
7	0.912	0.986	+0.095	0.4792
8	0.960	0.983	+0.196	0.4412
9	0.984	0.986	+0.291	0.4279
10	1.006	0.983	+0.391	0.4114
11	1.027	0.983	+0.490	0.3991
12	1.059	0.978	+0.587	0.3784
13	1.099	0.986	+0.688	0.3624
14	1.135	0.984	+0.784	0.3447
15	1.231	0.985	+0.977	0.3048
1	1.009	1.153	—0.585	0.5214
2	1.012	1.147	—0.408	0.5150
3	1.018	1.148	—0.201	0.5118
4	1.077	1.151	+0.001	0.4722
5	1.118	1.149	+0.192	0.4453
6	1.156	1.147	+0.388	0.4227
7	1.206	1.151	+0.585	0.3983
8	1.234	1.149	+0.688	0.3839
9	1.264	1.152	+0.783	0.3715
10	1.344	1.149	+0.980	0.3374
1	1.146	1.307	—0.401	0.5221
2	1.202	1.313	—0.201	0.4892
3	1.242	1.311	—0.006	0.4644
4	1.270	1.312	+0.189	0.4497
5	1.303	1.312	+0.387	0.4322
6	1.354	1.312	+0.595	0.4087
1	1.348	1.479	—0.395	0.4934
2	1.370	1.479	—0.192	0.4819
3	1.393	1.476	+0.004	0.4683

Series No. 122.



(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.355	0.331	—0.282	0.3885
2	0.358	0.330	—0.188	0.3845
3	0.354	0.327	—0.092	0.3827
4	0.362	0.329	+0.003	0.3758
5	0.361	0.327	+0.094	0.3729
6	0.370	0.331	+0.180	0.3654
7	0.382	0.327	+0.279	0.3420
8	0.441	0.330	+0.365	0.2792

(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.496	0.502	—0.373	0.4339
2	0.494	0.494	—0.284	0.4278
3	0.496	0.500	—0.184	0.4302
4	0.497	0.499	—0.091	0.4277
5	0.496	0.494	+0.005	0.4235
6	0.505	0.494	+0.095	0.4153
7	0.505	0.493	+0.181	0.4128
8	0.516	0.499	+0.280	0.4054
9	0.537	0.503	+0.370	0.3866
10	0.599	0.501	+0.466	0.3269
11	0.655	0.496	+0.561	0.2813
12	0.721	0.501	+0.649	0.2470
1	0.622	0.658	—0.181	0.4611
2	0.632	0.666	—0.091	0.4595
3	0.627	0.656	—0.002	0.4547
4	0.639	0.663	+0.090	0.4488
5	0.634	0.657	+0.179	0.4480
6	0.641	0.657	+0.269	0.4408
7	0.657	0.657	+0.368	0.4244
8	0.707	0.658	+0.467	0.3822
9	0.756	0.658	+0.557	0.3460
10	0.806	0.665	+0.650	0.3190
11	0.861	0.662	+0.742	0.2863
1	0.755	0.822	+0.001	0.4827
2	0.762	0.823	+0.097	0.4776
3	0.760	0.819	+0.179	0.4749
4	0.766	0.818	+0.272	0.4688
5	0.782	0.822	+0.368	0.4579
6	0.832	0.819	+0.469	0.4157
7	0.869	0.824	+0.559	0.3922
8	0.902	0.825	+0.649	0.3716
9	0.948	0.825	+0.742	0.3449
10	0.998	0.817	+0.843	0.3141
11	1.060	0.825	+0.928	0.2918
1	0.884	0.987	+0.182	0.5018
2	0.894	0.993	+0.277	0.4967
3	0.923	0.987	+0.369	0.4708
4	0.959	0.981	+0.462	0.4401
5	0.993	0.995	+0.560	0.4263
6	1.018	0.986	+0.648	0.4048
7	1.059	0.995	+0.743	0.3878
8	1.090	0.984	+0.838	0.3635
9	1.134	0.983	+0.922	0.3435
10	1.186	0.984	+1.016	0.3211
1	1.068	1.153	+0.365	0.4783
2	1.100	1.149	+0.466	0.4549
3	1.123	1.158	+0.557	0.4471
4	1.141	1.150	+0.651	0.4315
5	1.177	1.154	+0.743	0.4137
6	1.205	1.152	+0.839	0.3984
7	1.236	1.155	+0.923	0.3850
8	1.283	1.157	+1.018	0.3646
9	1.321	1.148	+1.112	0.3450
10	1.387	1.152	+1.208	0.3225
1	1.248	1.314	+0.559	0.4619
2	1.269	1.317	+0.653	0.4520
3	1.290	1.314	+0.746	0.4388
4	1.318	1.317	+0.835	0.4268
5	1.347	1.317	+0.923	0.4130
6	1.387	1.315	+1.025	0.3940
7	1.423	1.315	+1.110	0.3880
8	1.475	1.324	+1.203	0.3640
1	1.406	1.480	+0.655	0.4632
2	1.417	1.481	+0.745	0.4581
3	1.444	1.478	+0.831	0.4444
4	1.465	1.483	+0.926	0.4365
5	1.498	1.479	+1.018	0.4204
6	1.526	1.478	+1.110	0.4085
7	1.572	1.477	+1.202	0.3909

Mr. Nelles.

Bazin's Experiments.—Table showing the Heads corresponding to the same Discharge over the Four Weirs just considered, for Various Elevations of the Surface of the Water retained below the Weir.

A denotes weir 0.75 m. = 2.46 ft. high; with crest 0.20 m. = 0.66 ft. wide.
B " " 0.35 m. = 1.15 " " " " 0.20 m. = 0.66 " "
C denotes weir 0.75 m. = 2.46 ft. high; with crest 0.10 m. = 0.33 ft. wide
D " " 0.35 m. = 1.15 " " " " 0.10 m. = 0.33 " "

(1) Head on the standard weir of comparison (H) in feet.	(2) Description of weir under consideration.	(3) Head on similar weir when flow is not affected by water below.	Head, in Feet, on the Weirs under Consideration. Height of the water surface below the weirs, above the level of the crest h_1.													Remarks.
			0	0.098	0.197	0.295	0.394	0.492	0.591	0.689	0.787	0.886	0.984	1.083	1.181	
0.328.....	*A*	0.387	0.391	0.391	0.394	0.404	0.443	0.542	.....	.....	.....	.....	.....	.....	.....	The nappe was detached from the crest when head became about 0.4 ft.
	B	0.384	0.387	0.391	0.414	0.404	0.439	0.509	.....	.....	.....	.....	.....	.....	.....	
	C	0.358	0.361	0.364	0.371	0.384	0.456	0.456	.....	.....	.....	.....	.....	.....	.....	
	D	0.351	0.361	0.364	0.371	0.387	0.446	0.446	.....	.....	.....	.....	.....	.....	.....	
0.492.....	*A*	0.548	0.551	0.558	0.561	0.568	0.578	0.608	0.676	0.787	.....	.....	.....	.....	.....	The nappe was detached from crest when the head became about 0.53 ft.
	B	0.548	0.551	0.551	0.555	0.558	0.568	0.598	0.659	0.742	.....	.....	.....	.....	.....	
	C	0.499	0.502	0.509	0.518	0.532	0.598	0.598	0.598	0.598	.....	.....	.....	.....	.....	
	D	0.479	0.495	0.499	0.509	0.522	0.551	0.626	0.676	0.676	.....	.....	.....	.....	.....	
0.656.....	*A*	0.699	0.709	0.715	0.719	0.725	0.732	0.745	0.765	0.830	0.919	0.919	.....	.....	.....	The nappe was detached from the crest when the head became about 0.7 ft.
	B	0.689	0.689	0.692	0.699	0.706	0.709	0.719	0.745	0.794	0.866	0.948	.....	.....	.....	
	C	0.630	0.636	0.643	0.653	0.666	0.719	0.762	0.810	0.856	0.856	0.856	.....	.....	.....	
	D	0.614	0.626	0.633	0.636	0.643	0.673	0.715	0.768	0.820	0.820	0.820	.....	.....	.....	
0.820.....	*A*	0.830	0.853	0.856	0.859	0.866	0.876	0.883	0.896	0.926	0.981	1.060	1.145	1.145	.....	The nappe was detached from the crest when the head became about 0.8 ft.
	B	0.827	0.830	0.833	0.837	0.840	0.843	0.850	0.869	0.889	0.945	1.014	1.083	1.174	.....	
	C	0.752	0.775	0.784	0.794	0.817	0.830	0.893	0.929	0.977	1.027	1.027	1.027	1.027	.....	
	D	0.745	0.752	0.758	0.762	0.772	0.797	0.833	0.873	0.916	0.971	1.027	1.027	1.027	.....	
0.984.....	*A*	0.964	0.987	0.991	0.994	1.000	1.007	1.014	1.027	1.047	1.103	1.168	1.227	1.292	1.355	The nappe was detached from the crest when the head became about 0.95 ft.
	B	0.958	0.958	0.958	0.961	0.964	0.971	0.981	0.991	1.014	1.033	1.086	1.151	1.224	1.299	
	C	0.879	0.906	0.926	0.954	0.977	1.004	1.033	1.063	1.103	1.145	1.188	1.237	1.237	1.237	
	D	0.873	0.873	0.886	0.886	0.896	0.932	0.945	1.000	1.041	1.086	1.132	1.171	1.224	1.224	
1.148.....	*A*	1.099	1.119	1.126	1.129	1.132	1.142	1.145	1.161	1.178	1.214	1.276	1.342	1.414	1.414	The nappe was detached from the crest when the head became about 1.1 ft.
	B	1.096	1.096	1.096	1.096	1.096	1.099	1.109	1.116	1.129	1.145	1.174	1.227	1.289	1.361	
	C	1.017	1.083	1.093	1.116	1.136	1.151	1.178	1.207	1.234	1.267	1.309	1.355	1.355	1.355	
	D	1.080	1.080	1.080	1.080	1.080	1.073	1.099	1.126	1.155	1.184	1.221	1.257	1.302	1.361	
1.312.....	*A*	1.230	1.254	1.257	1.260	1.267	1.270	1.279	1.292	1.305	1.332	1.381	1.451	1.451	1.451	The nappe was detached from the crest when the head became about 1.2 ft.
	B	1.224	1.224	1.224	1.224	1.224	1.224	1.230	1.244	1.250	1.263	1.286	1.325	1.381	1.441	
	C	1.155	1.240	1.257	1.276	1.289	1.309	1.328	1.355	1.371	1.401	1.444	1.476	1.476	1.476	
	D	1.221	1.221	1.221	1.221	1.221	1.221	1.234	1.254	1.279	1.302	1.335	1.368	1.408	1.457	

Mr. Nelles. "In the experiments with weirs in thin partition, we have had recourse to Equations 11 and 12" (page 265), "to deduce the value of the coefficient m, but the presence of the sill introduces such complications that these equations are no longer perfectly applicable. The constants, which appear in the equations, vary so greatly with the head and width of crest, that it becomes impossible to establish a simple and practical formula.

"We must, therefore, content ourselves with a general *résumé* in a table showing the heads on the four weirs under consideration, corresponding to constant discharges and varying elevation of the water surface below the weirs. The corresponding head, where the flow is not affected by the retained water, is also shown."

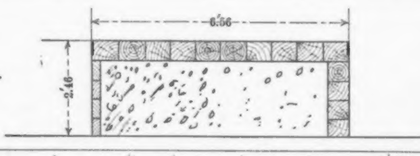
"We see that for the same discharge the heads vary notably from one weir to the other. The question is singularly complicated by the detachment of the nappe, which leaves the sill when the head attains certain limits depending on the width of the crest. The head and coefficient are then modified suddenly. This limit depends on the height of the retained water, and is not the same, even under fixed conditions. We cannot, therefore, establish as precise formulas as in the more simple case of the flow over weirs in thin partition."

Bazin's Experiments.—Series No. 124.

Nappe wetted underneath, with water retained in pool below weir. Height of weir under observation, 0.75 m. = 2.46 ft. Width of crest, 2.0 m. = 6.56 ft. Length, 2.0 m. = 6.56 ft.

Height of standard weir of comparison, 0.75 m. = 2.46 ft. Length, 2.0 m. = 6.56 ft.

Bazin's Series No. 124.



(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.398	0.325	+0.098	0.3199
2	0.403	0.330	+0.197	0.3227
3	0.399	0.325	+0.299	0.3187
4	0.451	0.328	+0.394	0.2678
5	0.526	0.326	+0.491	0.2111
1	0.598	0.494	+0.295	0.3228
2	0.590	0.492	+0.393	0.3264
3	0.609	0.492	+0.496	0.3114
4	0.668	0.490	+0.594	0.2684
5	0.748	0.489	+0.686	0.2263
1	0.777	0.655	+0.497	0.3301
2	0.781	0.657	+0.603	0.3288
3	0.811	0.656	+0.691	0.3099
4	0.890	0.658	+0.796	0.2709
5	0.961	0.660	+0.890	0.2422

(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.961	0.815	+0.593	0.2325
2	0.966	0.814	+0.692	0.3289
3	0.971	0.820	+0.792	0.3299
4	1.024	0.817	+0.890	0.3034
5	1.094	0.814	+0.989	0.2736
6	1.170	0.812	+1.084	0.2462
1	1.158	0.990	+0.794	0.3364
2	1.157	0.986	+0.895	0.3355
3	1.180	0.988	+0.993	0.3266
4	1.232	0.985	+1.091	0.3048
5	1.298	0.981	+1.183	0.2798
1	1.343	1.148	+0.985	0.3379
2	1.352	1.149	+1.078	0.3351
3	1.370	1.148	+1.180	0.3277
4	1.441	1.149	+1.276	0.3043

In his 6th and last article Bazin discusses the flow over submerged weirs, with sloping faces and crests of different widths and forms, and gives the results of 11 series, consisting of 724 individual observations. The methods used were similar to those already described for weirs of squared timbers (beam weirs). Bazin says of these experiments: Mr. Nelles.

"It now remains to terminate that which concerns the flow over weirs with sloping faces by describing the experiments made with water retained below the weirs.

"They form 11 series, three on weirs with sharp crest, seven on weirs with crest 0.20 m. (0.66 ft.) wide, one on weirs with crest 0.40 m. (1.32 ft.) wide."

BAZIN'S EXPERIMENTS.—SERIES NOS. 195 TO 197.

Nappe wetted underneath, with water retained in pool below weir.

Description of weirs under observation: Height, 0.75 m. = 2.46 ft. Length, 2.0 m. = 6.56 ft. Crest sharp. Upper face vertical, lower face sloping.

Dimensions of standard weir used for measuring the discharge: Height, 1.135 m. = 3.72 ft. Length, 2.0 m. = 6.56 ft.

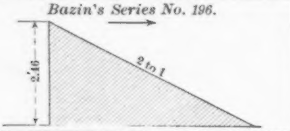
Bazin's Series No. 195.

(1) Number of experiment.	(2) Observed Heads in Feet. On the weir under consideration. (h)	(3) Observed Heads in Feet. On the standard weir of comparison. (H)	(4) Height of the retained water with reference to crest. + = above. − = below. (h_1)	(5) Coefficients of Discharge. For the Bazin formula. (m)
1	0.310	0.331	−0.196	0.4785
2	0.318	0.326	+0.007	0.4482
3	0.337	0.328	+0.103	0.4163
4	0.383	0.331	+0.198	0.3492
5	0.419	0.331	+0.296	0.3046
1	0.452	0.490	−0.396	0.4808
2	0.461	0.493	−0.194	0.4726
3	0.465	0.489	−0.004	0.4612
4	0.488	0.491	+0.100	0.4315
5	0.510	0.497	+0.198	0.4131
6	0.533	0.495	+0.199	0.3830
7	0.556	0.500	+0.297	0.3655
8	0.594	0.490	+0.403	0.3216
1	0.605	0.654	−0.592	0.4800
2	0.600	0.653	−0.391	0.4842
3	0.610	0.654	−0.200	0.4746
4	0.620	0.654	+0.006	0.4619
5	0.634	0.660	+0.100	0.4526
6	0.650	0.653	+0.203	0.4288
7	0.674	0.654	+0.294	0.4077
8	0.702	0.660	+0.297	0.3886
9	0.732	0.656	+0.403	0.3617
10	0.806	0.656	+0.591	0.3130
1	0.743	0.819	−0.793	0.4917
2	0.750	0.817	−0.393	0.4837
3	0.746	0.817	−0.196	0.4873
4	0.758	0.812	−0.002	0.4722
5	0.792	0.815	+0.198	0.4442
6	0.844	0.815	+0.295	0.4041
7	0.863	0.813	+0.392	0.3897
8	0.930	0.814	+0.596	0.3489
9	1.024	0.818	+0.793	0.3041
1	0.885	0.976	−0.788	0.4946
2	0.886	0.973	−0.385	0.4906
3	0.901	0.988	−0.199	0.4905
4	0.908	0.979	+0.002	0.4783
5	0.941	0.984	+0.208	0.4567
6	1.016	0.982	+0.402	0.4048

Mr. Nelles.

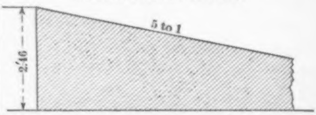
(1)	(2) h	(3) H	(4) h_1	(5) m
7	1.021	0.983	+0.402	0.4022
8	1.070	0.978	+0.600	0.3734
9	1.144	0.982	+0.795	0.3390
10	1.241	0.986	+0.986	0.3019
1	1.042	1.147	−0.582	0.4935
2	1.041	1.148	−0.389	0.4944
8	1.051	1.156	−0.197	0.4931
4	1.058	1.150	0.000	0.4848
5	1.095	1.152	+0.208	0.4614
6	1.171	1.149	+0.406	0.4154
7	1.212	1.144	+0.593	0.3919
8	1.217	1.146	+0.599	0.3904
9	1.280	1.156	+0.792	0.3664
10	1.355	1.149	+0.986	0.3336
1	1.189	1.317	−0.396	0.4994
2	1.190	1.311	−0.195	0.4958
3	1.226	1.313	+0.004	0.4753
4	1.242	1.313	+0.197	0.4658
5	1.309	1.307	+0.393	0.4274
6	1.358	1.312	+0.598	0.4070
7	1.412	1.314	+0.790	0.3847
1	1.336	1.474	−0.192	0.4970
2	1.345	1.475	+0.004	0.4926
3	1.409	1.474	+0.204	0.4599
4	1.449	1.467	+0.388	0.4371

Bazin's Series No. 196.



(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.327	0.326	−0.200	0.4312
2	0.328	0.325	−0.006	0.4286
3	0.342	0.326	+0.097	0.4036
4	0.360	0.323	+0.189	0.3702
5	0.407	0.330	+0.289	0.3167
6	0.468	0.323	+0.392	0.2486
1	0.484	0.486	−0.200	0.4319
2	0.485	0.491	−0.001	0.4353
3	0.495	0.491	+0.097	0.4227
4	0.507	0.492	+0.189	0.4081
5	0.533	0.487	+0.292	0.3753
6	0.570	0.491	+0.392	0.3423
7	0.637	0.487	+0.493	0.2857
1	0.645	0.656	+0.001	0.4364
2	0.655	0.649	+0.194	0.4202
3	0.664	0.654	+0.207	0.4162
4	0.680	0.653	+0.296	0.4004
5	0.700	0.653	+0.395	0.3838
6	0.743	0.659	+0.492	0.3558
7	0.801	0.654	+0.589	0.3138
8	0.865	0.657	+0.688	0.2816
1	0.804	0.819	0.000	0.4371
2	0.812	0.819	+0.197	0.4316
3	0.824	0.820	+0.287	0.4215
4	0.841	0.817	+0.385	0.4078
5	0.873	0.818	+0.491	0.3863
6	0.902	0.819	+0.590	0.3676
7	0.982	0.820	+0.693	0.3240
8	1.029	0.819	+0.781	0.3020
1	0.962	0.985	+0.198	0.4414
2	0.985	0.982	+0.392	0.4235
3	1.005	0.985	+0.493	0.4126
4	1.029	0.981	+0.586	0.3963
5	1.065	0.984	+0.691	0.3789
6	1.146	0.983	+0.785	0.3383
7	1.196	0.983	+0.886	0.3175
8	1.239	0.981	+0.983	0.2998
1	1.119	1.152	+0.195	0.4460
2	1.131	1.150	+0.386	0.4374
3	1.147	1.143	+0.490	0.4248
4	1.164	1.146	+0.594	0.4153
5	1.199	1.146	+0.697	0.3986
6	1.230	1.149	+0.789	0.3851
7	1.281	1.150	+0.893	0.3631
8	1.358	1.147	+0.980	0.3322
9	1.414	1.153	+1.091	0.3144
1	1.281	1.313	+0.389	0.4443
2	1.311	1.311	+0.590	0.4284
3	1.330	1.306	+0.682	0.4165
4	1.357	1.311	+0.783	0.4065
5	1.390	1.313	+0.880	0.3928
6	1.434	1.314	+0.977	0.3757

Bazin's Series No. 197.



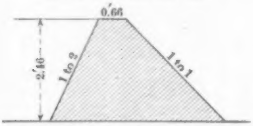
(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.355	0.325	+0.195	0.3793
2	0.384	0.325	+0.295	0.3373
3	0.454	0.326	+0.397	0.2630
4	0.595	0.327	+0.494	0.2102
1	0.527	0.491	+0.294	0.3859
2	0.552	0.494	+0.397	0.3620
3	0.595	0.491	+0.493	0.3210
4	0.675	0.491	+0.594	0.2655
5	0.748	0.490	+0.691	0.2268
1	0.698	0.655	+0.403	0.3876
2	0.711	0.655	+0.494	0.3756
3	0.750	0.652	+0.589	0.3413
4	0.792	0.653	+0.694	0.3185
5	0.894	0.653	+0.795	0.2663
6	0.967	0.655	+0.887	0.2369
1	0.862	0.815	+0.494	0.3919
2	0.876	0.812	+0.596	0.3807
3	0.910	0.812	+0.693	0.3594
4	0.956	0.810	+0.791	0.3317
5	1.018	0.818	+0.888	0.3067
6	1.047	0.809	+0.993	0.2639
7	1.192	0.815	+1.092	0.2409
1	1.038	0.985	+0.600	0.3946
2	1.042	0.976	+0.690	0.3874
3	1.077	0.986	+0.792	0.3741
4	1.113	0.977	+0.885	0.3513
5	1.170	0.983	+0.988	0.3278
6	1.251	0.981	+1.084	0.2967
7	1.323	0.986	+1.176	0.2744
1	1.216	1.149	+0.792	0.3921
2	1.232	1.140	+0.882	0.3798
3	1.278	1.146	+0.995	0.3622
4	1.326	1.145	+1.091	0.3424
5	1.372	1.146	+1.180	0.3256
6	1.462	1.146	+1.279	0.2962
1	1.374	1.303	+0.882	0.3956
2	1.404	1.311	+0.985	0.3861
3	1.429	1.304	+1.082	0.3735
4	1.478	1.311	+1.182	0.3563

BAZIN'S EXPERIMENTS.—SERIES NOS. 198 TO 208.

Mr. Nelles

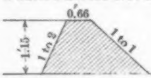
Nappe wetted underneath, with water retained in pool below weir. Description of weirs under observation: Height 0.75 m. = 2.46 ft. 0.35 m. = 1.15 ft. Length 2.0 m. = 6.56 ft. Crest 0.20 m. = 0.66 ft. wide. Upper and lower faces inclined.

Bazin's Series No. 198.



(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.370	0.327	+0.098	0.3602
2	0.380	0.332	+0.144	0.3549
3	0.379	0.336	+0.147	0.3600
4	0.376	0.328	+0.197	0.3529
5	0.383	0.326	+0.295	0.3402
6	0.431	0.328	+0.394	0.2856
7	0.520	0.328	+0.492	0.2156
1	0.543	0.497	+0.197	0.3757
2	0.536	0.496	+0.198	0.3804
3	0.539	0.495	+0.249	0.3758
4	0.553	0.501	+0.289	0.3650
5	0.559	0.495	+0.403	0.3559
6	0.590	0.496	+0.492	0.3305
7	0.661	0.492	+0.592	0.2743
1	0.682	0.656	+0.198	0.4007
2	0.711	0.660	+0.395	0.3802
3	0.691	0.657	+0.397	0.3933
4	0.698	0.656	+0.438	0.3872
5	0.723	0.661	+0.509	0.3717
6	0.743	0.658	+0.589	0.3546
7	0.806	0.655	+0.687	0.3115
8	0.895	0.657	+0.789	0.2675
9	0.973	0.659	+0.888	0.2372
1	0.825	0.823	+0.299	0.4231
2	0.841	0.819	+0.395	0.4161
3	0.854	0.823	+0.488	0.4018
4	0.839	0.818	+0.491	0.4088
5	0.864	0.824	+0.543	0.3962
6	0.845	0.822	+0.544	0.4074
7	0.868	0.818	+0.591	0.3888
8	0.858	0.822	+0.592	0.3982
9	0.888	0.818	+0.688	0.3756
10	0.963	0.820	+0.788	0.3331
11	1.035	0.823	+0.879	0.3015
12	1.115	0.822	+0.994	0.2694
1	0.954	0.981	+0.300	0.4432
2	0.954	0.976	+0.395	0.4401
3	0.962	0.980	+0.490	0.4369
4	0.973	0.986	+0.592	0.4335
5	0.995	0.982	+0.598	0.4174
6	0.985	0.987	+0.635	0.4266
7	1.011	0.986	+0.692	0.4103
8	1.039	0.985	+0.789	0.3929
9	1.119	0.988	+0.884	0.3536
10	1.196	0.985	+0.983	0.3179
11	1.192	0.985	+0.987	0.3193
12	1.258	0.989	+1.083	0.2969
1	1.092	1.148	+0.397	0.4601
2	1.092	1.144	+0.496	0.4575
3	1.105	1.149	+0.593	0.4523
4	1.115	1.148	+0.689	0.4458
5	1.137	1.144	+0.691	0.4307
6	1.131	1.147	+0.745	0.4355
7	1.153	1.148	+0.788	0.4231
8	1.189	1.149	+0.884	0.4057
9	1.281	1.148	+0.979	0.3612
10	1.354	1.146	+1.087	0.3378
11	1.410	1.148	+1.182	0.3131
1	1.226	1.312	+0.494	0.4738
2	1.227	1.310	+0.593	0.4714
3	1.239	1.313	+0.689	0.4664
4	1.268	1.311	+0.740	0.4501
5	1.248	1.313	+0.741	0.4620
6	1.281	1.313	+0.785	0.4433
7	1.259	1.311	+0.786	0.4541
8	1.288	1.310	+0.835	0.4383
9	1.291	1.316	+0.841	0.4397
10	1.306	1.314	+0.888	0.4314
11	1.373	1.314	+0.983	0.4001
12	1.446	1.309	+1.083	0.3685
1	1.362	1.475	+0.594	0.4816
2	1.376	1.471	+0.784	0.4744
3	1.424	1.477	+0.887	0.4529
4	1.418	1.476	+0.887	0.4560

Bazin's Series No. 199.

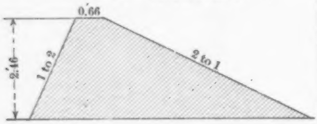


(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.375	0.327	−0.190	0.3516
2	0.376	0.331	+0.001	0.3560
3	0.378	0.327	+0.178	0.3474
4	0.380	0.326	+0.276	0.3446
5	0.419	0.330	+0.376	0.3033
6	0.489	0.320	+0.460	0.2300
7	0.581	0.332	+0.561	0.1868
1	0.536	0.500	+0.002	0.3855
2	0.535	0.493	+0.177	0.3779
3	0.537	0.497	+0.273	0.3800
4	0.555	0.501	+0.366	0.3665
5	0.566	0.490	+0.460	0.3453
6	0.622	0.495	+0.565	0.3031
7	0.696	0.493	+0.649	0.2552
1	0.670	0.655	+0.002	0.4101
2	0.676	0.656	+0.176	0.4060
3	0.674	0.656	+0.273	0.4084
4	0.688	0.656	+0.373	0.3950
5	0.697	0.651	+0.466	0.3842
6	0.726	0.656	+0.565	0.3651
7	0.748	0.656	+0.644	0.3491

Mr. Nelles.

(1)	(2) h	(3) H	(4) h_1	(5) m
8	0.824	0.657	+0.745	0.3023
9	0.903	0.655	+0.838	0.2627
10	0.989	0.660	+0.933	0.2314
1	0.806	0.821	+0.095	0.4370
2	0.799	0.811	+0.271	0.4344
3	0.809	0.810	+0.367	0.4259
4	0.827	0.821	+0.464	0.4202
5	0.834	0.823	+0.504	0.4161
6	0.839	0.821	+0.519	0.4103
7	0.848	0.820	+0.568	0.4035
8	0.855	0.815	+0.681	0.3949
9	0.880	0.813	+0.736	0.3771
10	0.947	0.817	+0.837	0.3401
11	1.018	0.811	+0.923	0.3019
12	1.094	0.814	+1.013	0.2725
1	0.932	0.983	+0.279	0.4608
2	0.941	0.985	+0.367	0.4557
3	0.944	0.980	+0.465	0.4495
4	0.960	0.980	+0.557	0.4382
5	0.976	0.981	+0.646	0.4281
6	0.993	0.976	+0.740	0.4153
7	1.019	0.979	+0.837	0.4003
8	1.082	0.978	+0.932	0.3659
9	1.146	0.976	+1.016	0.3350
10	1.220	0.979	+1.108	0.3057
11	1.305	0.978	+1.212	0.2758
1	1.064	1.150	+0.370	0.4789
2	1.070	1.147	+0.461	0.4731
3	1.085	1.159	+0.517	0.4710
4	1.080	1.147	+0.555	0.4668
5	1.098	1.149	+0.653	0.4559
6	1.111	1.149	+0.735	0.4480
7	1.135	1.150	+0.835	0.4347
8	1.154	1.151	+0.921	0.4357
9	1.200	1.143	+1.015	0.3955
10	1.274	1.146	+1.108	0.3636
11	1.360	1.150	+1.208	0.3316
1	1.190	1.309	+0.461	0.4934
2	1.209	1.318	+0.563	0.4865
3	1.213	1.305	+0.647	0.4770
4	1.223	1.293	+0.749	0.4648
5	1.253	1.314	+0.842	0.4594
6	1.272	1.316	+0.934	0.4499
7	1.284	1.315	+1.012	0.4426
8	1.341	1.320	+1.113	0.4172
9	1.420	1.311	+1.206	0.3792
1	1.339	1.469	+0.655	0.4970
2	1.349	1.478	+0.736	0.4911
3	1.355	1.475	+0.837	0.4859
4	1.373	1.475	+0.917	0.4765
5	1.389	1.474	+1.019	0.4679
6	1.419	1.476	+1.115	0.4545
7	1.474	1.476	+1.205	0.4298
8	1.531	1.472	+1.291	0.4036

Bazin's Series No. 200.

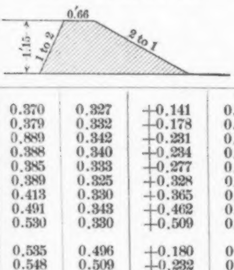


(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.386	0.335	+0.093	0.3480
2	0.385	0.330	+0.189	0.3461
3	0.387	0.334	+0.234	0.3456
4	0.383	0.329	+0.235	0.3448
5	0.387	0.329	+0.282	0.3382
6	0.392	0.326	+0.292	0.3278
7	0.405	0.331	+0.331	0.3202
8	0.437	0.332	+0.391	0.2850
9	0.518	0.326	+0.494	0.2146
10	0.616	0.330	+0.591	0.1696
1	0.550	0.502	+0.092	0.3724
2	0.544	0.497	+0.189	0.3726
3	0.544	0.494	+0.255	0.3701
4	0.544	0.492	+0.291	0.3679
5	0.557	0.496	+0.384	0.3595
6	0.561	0.502	+0.387	0.3610
7	0.567	0.494	+0.435	0.3473
8	0.569	0.493	+0.437	0.3447
9	0.569	0.497	+0.457	0.3489
10	0.590	0.496	+0.490	0.3299
11	0.584	0.495	+0.494	0.3333
12	0.602	0.499	+0.533	0.3229
13	0.666	0.496	+0.587	0.2743
14	0.749	0.495	+0.682	0.2294
15	0.839	0.491	+0.787	0.1908
1	0.699	0.661	+0.189	0.3904
2	0.703	0.664	+0.285	0.3901
3	0.704	0.661	+0.386	0.3865
4	0.714	0.659	+0.487	0.3760
5	0.722	0.666	+0.539	0.3756
6	0.728	0.657	+0.566	0.3645
7	0.733	0.655	+0.590	0.3590
8	0.734	0.659	+0.590	0.3610
9	0.751	0.658	+0.634	0.3485
10	0.759	0.657	+0.685	0.3421
11	0.802	0.657	+0.687	0.3144
12	0.894	0.655	+0.789	0.2662
13	0.970	0.655	+0.886	0.2351
14	1.050	0.657	+0.977	0.1799
1	0.840	0.821	+0.187	0.4102
2	0.837	0.820	+0.294	0.4112
3	0.837	0.815	+0.389	0.4080
4	0.842	0.815	+0.491	0.4035
5	0.852	0.815	+0.589	0.3977
6	0.869	0.812	+0.687	0.3818
7	0.880	0.819	+0.686	
8	0.888	0.817	+0.731	0.3744
9	0.903	0.819	+0.741	0.3665
10	0.948	0.823	+0.787	0.3433
11	0.908	0.819	+0.788	0.3629
12	0.931	0.818	+0.821	0.3492
13	1.002	0.814	+0.842	0.3103
14	1.032	0.809	+0.883	0.2944
15	1.115	0.815	+0.979	0.2653
16	1.199	0.820	+1.092	0.2398
17	1.274	0.819	+1.183	0.2188
1	0.978	0.982	+0.290	0.4279
2	0.978	0.979	+0.387	0.4252
3	0.983	0.982	+0.493	0.4244
4	0.987	0.980	+0.588	0.4199
5	0.997	0.977	+0.686	0.4124
6	1.012	0.976	+0.785	0.4023
7	1.031	0.982	+0.826	0.3947
8	1.041	0.978	+0.842	0.3870
9	1.044	0.974	+0.880	0.3828
10	1.082	0.982	+0.881	0.3672
11	1.067	0.979	+0.925	0.3732
12	1.144	0.978	+0.933	0.3358
13	1.193	0.976	+0.983	0.3144
14	1.236	0.971	+1.048	0.2959
15	1.262	0.982	+1.077	0.2913
16	1.339	0.979	+1.176	0.2655

Mr. Nelles.

(1)	(2) h	(3) H	(4) h_1	(5) m
1	1.120	1.148	+0.295	0.4422
2	1.118	1.148	+0.389	0.4432
3	1.119	1.146	+0.509	0.4408
4	1.125	1.148	+0.590	0.4385
5	1.123	1.141	+0.689	0.4356
6	1.144	1.149	+0.781	0.4286
7	1.158	1.141	+0.884	0.4159
8	1.173	1.143	+0.928	0.4093
9	1.236	1.141	+0.983	0.3769
10	1.189	1.140	+0.984	0.3992
11	1.190	1.136	+1.032	0.3516
12	1.209	1.135	+1.039	0.3865
13	1.240	1.147	+1.067	0.3787
14	1.337	1.146	+1.075	0.3377
15	1.410	1.148	+1.181	0.3125
1	1.250	1.300	+0.394	0.4528
2	1.250	1.296	+0.487	0.4507
3	1.254	1.297	+0.591	0.4496
4	1.256	1.301	+0.682	0.4501
5	1.263	1.297	+0.786	0.4440
6	1.274	1.296	+0.888	0.4380
7	1.299	1.297	+0.951	0.4261
8	1.325	1.296	+0.981	0.4128
9	1.379	1.296	+1.081	0.3892
10	1.443	1.296	+1.084	0.3636
11	1.453	1.300	+1.133	0.4011
12	1.466	1.305	+1.174	0.3844

Bazin's Series No. 201.



(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.370	0.327	+0.141	0.3600
2	0.379	0.332	+0.178	0.3566
3	0.889	0.342	+0.231	0.3560
4	0.388	0.340	+0.234	0.3532
5	0.385	0.333	+0.277	0.3462
6	0.389	0.325	+0.328	0.3297
7	0.413	0.330	+0.365	0.3085
8	0.491	0.343	+0.462	0.2522
9	0.530	0.330	+0.509	0.2119
1	0.535	0.496	+0.180	0.3817
2	0.548	0.509	+0.232	0.3822
3	0.442	0.501	+0.277	0.3799
4	0.539	0.494	+0.326	0.3747
5	0.546	0.502	+0.364	0.3765
6	0.551	0.501	+0.418	0.3691
7	0.547	0.492	+0.421	0.3644
8	0.558	0.491	+0.461	0.3525
9	0.584	0.501	+0.512	0.3384
10	0.622	0.502	+0.565	0.3094
11	0.709	0.500	+0.657	0.2527
12	0.791	0.495	+0.754	0.2112
1	0.687	0.666	+0.230	0.4051
2	0.692	0.669	+0.274	0.4034
3	0.684	0.656	+0.320	0.3987
4	0.682	0.659	+0.365	0.4037
5	0.692	0.661	+0.461	0.3964
6	0.694	0.650	+0.516	0.3856
7	0.712	6.655	+0.562	0.3744
8	0.708	0.654	+0.565	0.3772
9	0.726	0.660	+0.613	0.3687
10	0.730	0.665	+0.614	0.3692
11	0.755	0.661	+0.661	0.3480
12	0.784	0.661	+0.700	0.3283
13	0.832	0.665	+0.748	0.3081
14	0.914	0.664	+0.849	0.2632
15	0.992	0.663	+0.940	0.2324
1	0.815	0.818	+0.273	0.4262
2	0.819	0.819	+0.327	0.4249
3	0.817	0.819	+0.367	0.4265
4	0.815	0.812	+0.469	0.4221
5	0.839	0.827	+0.561	0.4150
6	0.847	0.820	+0.656	0.4041
7	0.855	0.817	+0.705	0.3964
8	0.854	0.810	+0.711	0.3923
9	0.875	0.823	+0.747	0.3877
10	0.869	0.810	+0.747	0.3816
11	0.910	0.829	+0.790	0.3694
12	0.950	0.819	+0.842	0.3395
13	1.041	0.821	+0.934	0.2973
14	1.077	0.817	+0.984	0.2800
15	1.111	0.819	+1.026	0.2686
16	1.190	0.812	+1.126	0.2396
1	0.950	0.981	+0.325	0.4457
2	0.947	0.979	+0.371	0.4463
3	0.954	0.988	+0.460	0.4481
4	0.952	0.976	+0.507	0.4412
5	0.954	0.983	+0.559	0.4442
6	0.964	0.984	+0.663	0.4382
7	0.956	0.974	+0.746	0.4269
8	0.989	0.982	+0.792	
9	0.994	0.989	+0.794	0.4219
10	0.991	0.986	+0.791	
11	1.001	0.976	+0.839	0.4095
12	1.006	0.983	+0.848	0.4102
13	1.035	0.986	+0.884	0.3955
14	1.024	0.990	+0.888	0.4046
15	1.077	0.987	+0.931	0.3739
16	1.075	0.988	+0.933	
17	1.121	0.983	+0.983	0.3335
18	1.168	0.979	+1.030	0.3269
19	1.237	0.984	+1.114	0.3016
20	1.320	0.983	+1.219	0.2732
1	1.080	1.146	+0.467	0.4661
2	1.085	1.143	+0.561	0.4602
3	1.085	1.146	+0.658	0.4624
4	1.093	1.141	+0.755	0.4541
5	1.108	1.145	+0.837	0.4474
6	1.126	1.152	+0.885	0.4419
7	1.129	1.142	+0.936	0.4334
8	1.158	1.153	+0.991	0.4234
9	1.164	1.151	+0.992	0.4192
10	1.205	1.146	+1.029	0.3953
11	1.244	1.150	+1.071	0.3789
12	1.292	1.144	+1.118	0.3546
13	1.373	1.146	+1.222	0.3246
1	1.204	1.300	+0.567	0.4791
2	1.212	1.306	+0.655	0.4781
3	1.223	1.313	+0.739	0.4757
4	1.221	1.299	+0.841	0.4690
5	1.234	1.303	+0.939	0.4634
6	1.244	1.293	+0.987	0.4531
7	1.261	1.305	+1.035	0.4503
8	1.281	1.305	+1.077	0.4393
9	1.329	1.303	+1.120	0.4146
10	1.377	1.303	+1.170	0.3934
11	1.428	1.294	+1.222	0.3684
12	1.459	1.297	+1.264	0.3588
13	1.503	1.304	+1.314	0.3455
1	1.333	1.461	+0.657	0.4911
2	1.335	1.461	+0.742	0.4905

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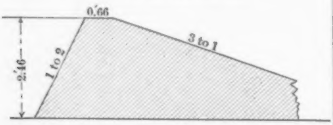
(1)	(2) h	(3) H	(4) h_1	(5) m
3	1.337	1.460	+0.842	0.4892
4	1.343	1.458	+0.933	0.4846
5	1.370	1.470	+1.029	0.4763
6	1.394	1.468	+1.119	0.4626
7	1.405	1·462	+1.171	0.4546
8	1.433	1.462	+1.212	0.4419
9	1.494	1.465	+1.263	0.4153
10	1.541	1.464	+1.313	0.3961

Bazin's Series No. 202.



(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.380	0.330	+0.192	0.3523
2	0.383	0.328	+0.290	0.3442
3	0.408	0.339	+0.345	0.3280
4	0.427	0.330	+0.394	0.2945
5	0.471	0.336	+0.439	0.2602
6	0.513	0.329	+0.487	0.2227
1	0.548	0.494	+0.290	0.3674
2	0.552	0.493	+0.389	0.3614
3	0.570	0.493	+0.487	0.3447
4	0.610	0.503	+0.547	0.3211
5	0.646	0.504	+0.598	0.2952
6	0.684	0.506	+0.637	0.2725
7	0.731	0.500	+0.687	0.2422
1	0.703	0.656	+0.286	0.3838
2	0.704	0.657	+0.388	0.3833
3	0.707	0.655	+0.479	0.3792
4	0.738	0.665	+0.588	0.3644
5	0.756	0.656	+0.673	0.3448
6	0.832	0.666	+0.748	0.3048
7	0.861	0.661	+0.794	0.2865
8	0.923	0.665	+0.842	0.2609
9	0.946	0.663	+0.882	0.2498
1	0.855	0.824	—0.190	0.4024
2	0.853	0.820	+0.006	0.4018
3	0.853	0.823	+0.200	0.4040
4	0.854	0.822	+0.395	0.4017
5	0.858	0.824	+0.485	0.4006
6	0.870	0.827	+0.600	0.3943
7	0.884	0.823	+0.686	0.3828
8	0.910	0.825	+0.788	0.3679
9	0.978	0.820	+0.869	0.3268
10	1.088	0.824	+0.983	0.2808
11	1.176	0.832	+1.092	0.2529
1	1.001	0.987	+0.282	0.4176
2	1.002	0.986	+0.388	0.4159
3	1.002	0.984	+0.477	0.4151
4	1.012	0.993	+0.584	0.4141
5	1.010	0.983	+0.671	0.4086
6	1.026	0.989	+0.787	0.4035
7	1.055	0.991	+0.877	0.3877
8	1.092	0.991	+0.966	0.3693
9	1.109	0.988	+0.987	0.3590
10	1.198	0.991	+1.064	0.3203
11	1.283	0.992	+1.184	0.2897
12	1.375	0.993	+1.269	0.2617
1	1.152	1.150	+0.491	0.4256
2	1.151	1.150	+0.590	0.4271
3	1.151	1.156	+0.687	0.4249
4	1.171	1.161	+0.783	0.4224
5	1.182	1.151	+0.897	0.4107
6	1.201	1.154	+0.975	0.4028
7	1.247	1.152	+1.091	0.3795
8	1.322	1.158	+1.168	0.3504
9	1.454	1.157	+1.286	0.3035
1	1.295	1.317	+0.590	0.4392
2	1.295	1.314	+0.688	0.4379
3	1.302	1.317	+0.784	0.4357
4	1.311	1.316	+0.885	0.4310
5	1.326	1.320	+0.976	0.4255
6	1.342	1.314	+1.084	0.4156
7	1.385	1.323	+1.180	0.3998
8	1.443	1.316	+1.279	0.3738

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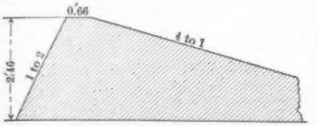
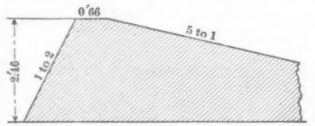
Bazin's Series No. 203.



(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.380	0.329	+0.186	0.3505
2	0.380	0.327	+0.284	0.3465
3	0.391	0.329	+0.332	0.3356
4	0.417	0.327	+0.385	0.3013
5	0.453	0.328	+0.431	0.2656
6	0.501	0.326	+0.484	0.2277
1	0.546	0.494	+0.291	0.3695
2	0.544	0.490	+0.388	0.3661
3	0.556	0.490	+0.485	0.3539
4	0.580	0.492	+0.533	0.3351
5	0.619	0.488	+0.584	0.2992
6	0.668	0.489	+0.632	0.2685
7	0.714	0.492	+0.684	0.2449
1	0.707	0.655	+0.289	0.3804
2	0.716	0.661	+0.380	0.3781
3	0.705	0.656	+0.391	0.3829
4	0.713	0.657	+0.491	0.3768
5	0.717	0.657	+0.588	0.3734
6	0.754	0.658	+0.684	0.3482
7	0.798	0.653	+0.730	0.3152
8	0.835	0.658	+0.780	0.2974
9	0.885	0.659	+0.831	0.2743
10	0.930	0.656	+0.886	0.2523
1	0.856	0.821	+0.293	0.3996
2	0.861	0.823	+0.393	0.3985
3	0.852	0.815	+0.490	0.3987
4	0.870	0.825	+0.591	0.3930
5	0.874	0.820	+0.683	0.3870
6	0.899	0.825	+0.785	0.3746
7	0.949	0 819	+0.881	0.3416
8	1.049	0.825	+0.981	0.2969
9	1.147	0.819	+1.078	0.2568
1	1.016	0.988	+0.482	0.4092
2	1.015	0.984	+0.592	0.4075
3	1.019	0.983	+0.680	0.4043
4	1.026	0.983	+0.780	0.4001
5	1.045	0.982	+0.878	0.3886
6	1.085	0.985	+0.983	0.3690
7	1.166	0.984	+1.072	0.3306
8	1.284	0.985	+1.182	0.2864
9	1.365	0.885	+1.274	0.2612
1	1.170	1.151	+0.578	0.4170
2	1.158	1.141	+0.689	0.4173
3	1.179	1.154	+0.775	0.4137
4	1.180	1.150	+0.881	0.4108
5	1.196	1.150	+0.976	0.4030
6	1.222	1.147	+1.072	0.3888
7	1.288	1.150	+1.176	0.3603
8	1.330	1.150	+1.273	0.3428
1	1.321	1.315	+0.682	0.4256
2	1.322	1.313	+0.784	0.4242
3	1.326	1.313	+0.879	0.4223
4	1.337	1.316	+0.980	0.4182
5	1.348	1.316	+1.068	0.4134
6	1.382	1.318	+1.181	0.3990
7	1.422	1.314	+1.278	0.3806

Bazin's Series No. 204.



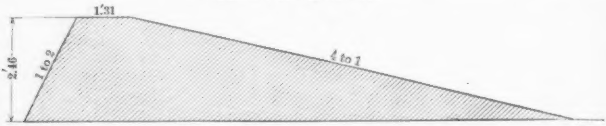
(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.384	0.330	+0.186	0.3459
2	0.383	0.329	+0.294	0.3457
3	0.390	0.330	+0.338	0.3380
4	0.430	0.329	+0.389	0.2897
5	0.450	0.330	+0.435	0·2718
6	0.510	0.328	+0.493	0.2237
1	0.545	0.487	+0.287	0.3620
2	0.556	0.495	+0.394	0.3609
3	0.556	0.489	+0.487	0.3537
4	0.582	0.490	+0.535	0.3310
5	0.588	0.496	+0.538	0.3326
6	0.622	0.486	+0.589	0.2966
7	0.658	0.492	+0.632	0.2764
8	0.720	0.486	+0.681	0.2380
1	0.714	0.659	+0.494	0.3774
2	0.723	0.657	+0.587	0.3693
3	0.765	0.658	+0.671	0.3410
4	0.786	0.662	+0.734	0.3288
5	0.833	0.658	+0.775	0.2992
6	0.880	0.663	+0.830	0.2785
7	0.929	0.676	+0.890	0.2642
1	0.871	0.827	+0.297	0.3930
2	0.870	0.822	+0.383	0.3903
3	0.883	0.833	+0.492	0.3901
4	0.875	0.822	+0.579	0.3873
5	0.894	0.836	+0.692	0.3848
6	0.900	0.822	+0.779	0.3711
7	0.962	0.832	+0.891	0.3422
8	1.041	0.822	+0.969	0.2988
9	1.145	0.832	+1.086	0.2638
1	1.026	0.986	+0.586	0.4022
2	1.025	0.984	+0.688	0.4012
3	1.025	0.978	+0.786	0.3981
4	1.049	0.982	+0.885	0.3866
5	1.070	0.978	+0.978	0.3731
6	1.158	0.983	+1.077	0.3334
7	1.260	0.983	+1.172	0.2940
8	1.349	0.982	+1.273	0.2647
1	1.175	1.148	+0.591	0.4129
2	1.183	1.147	+0.686	0.4080
3	1.182	1.150	+0.784	0.4101
4	1.188	1.144	+0.879	0.4041
5	1.211	1.153	+0.986	0.3971
6	1.227	1.150	+1.072	0.3880
7	1.276	1.146	+1.180	0.3632
8	1.380	1.148	+1.270	0.3243
1	1.329	1.310	+0.689	0.4190
2	1.338	1.313	+0.782	0.4163
3	1.337	1.312	+0.887	0.4166
4	1.347	1.312	+0.976	0.4119
5	1.353	1.305	+1.078	0.4065
6	1.380	1.313	+1.175	0.3975
7	1.421	1.310	+1.276	0.3791

Mr. Nelles.

Bazin's Experiments.—Series No. 205.

Nappe wetted underneath, with water retained in pool below weir. Description of weir under observation: Height, 0.75 m. = 2.46 ft. Length, 2.0 m. = 6.56 ft. Crest, 0.40 m. = 1.31 ft. wide. Both faces inclined.

Bazin's Series No. 205.



(1)	(2) h	(3) H	(4) h_1	(5) m	(1)	(2) h	(3) H	(4) h_1	(5) m
1	0.385	0.330	+0.305	0.3438	5	0.922	0.815	+0.788	0.3544
2	0.393	0.332	+0.354	0.3368	6	0.956	0.819	+0.884	0.3375
3	0.418	0.328	+0.393	0.3010	7	1.029	0.814	+0.986	0.2997
4	0.461	0.327	+0.441	0.2501	8	1.133	0.819	+1.084	0.2619
5	0.512	0.329	+0.491	0.2230	9	1.226	0.814	+1.183	0.2309
1	0.561	0.491	+0.386	0.3505	1	1.075	0.986	+0.694	0.3752
2	0.570	0.493	+0.491	0.3444	2	1.075	0.983	+0.782	0.3733
3	0.593	0.493	+0.538	0.3240	3	1.091	0.988	+0.886	0.3681
4	0.622	0.496	+0.593	0.3045	4	1.106	0.984	+0.975	0.3581
5	0.675	0.493	+0.642	0.2662	5	1.159	0.987	+1.081	0.3355
6	0.709	0.495	+0.689	0.2503	6	1.247	0.985	+1.178	0.2997
					7	1.345	0.987	+1.286	0.2684
1	0.749	0.655	+0.591	0.3482					
2	0.749	0.660	+0.639	0.3531	1	1.238	1.147	+0.886	0.3813
3	0.768	0.652	+0.691	0.3335	2	1.254	1.151	+0.981	0.3767
4	0.794	0.658	+0.741	0.3220	3	1.258	1.143	+1.081	0.3697
5	0.834	0.657	+0.786	0.2980	4	1.292	1.146	+1.179	0.3568
6	0.877	0.660	+0.839	0.2789	5	1.360	1.144	+1.277	0.3292
7	0.922	0.657	+0.887	0.2568					
					1	1.399	1.311	+0.985	0.3891
1	0.900	0.816	+0.394	0.3681	2	1.413	1.311	+1.088	0.3830
2	0.900	0.818	+0.497	0.3695	3	1.426	1.312	+1.182	0.3783
3	0.904	0.816	+0.590	0.3649	4	1.449	1.312	+1.279	0.3694
4	0.908	0.822	+0.688	0.3669					

"We have already shown, in discussing the similar experiments on beam weirs, that the size of the crest completely modifies the effect of the retained water. Thus, for weirs in thin partition, the effect of the retained water is manifest before it reaches the level of the crest, while for weirs with wide crest, the effect is not noticeable until the level of the retained water is above the crest. In order to show the results of the preceding experiments in a comprehensive manner, we have prepared the following tables, which show for the same discharge and for varying elevations of the retained water, the corresponding heads on the weirs under experiment.

"Notwithstanding some anomalies, the tables show that the influence of the retained water is hardly sensible so long as its height above the crest h_1 is less than one-half the head h. When we consider Series 205, in which the crest is 0.40 m. (1.31 ft.) wide, it is seen that the influence of the retained water is not appreciable until it attains a height of two-thirds of the head."

Bazin's Experiments.

Mr. Nelles.

Table Showing the Heads Corresponding to the same Discharge over Weirs with Sharp Crests and Sloping Faces, for Various Elevations of the Surface of the Water Retained Below Weir.

Head on standard weir of comparison. (H.)	Head when the discharge is not affected by the retained water.	Head, in Feet, on the Weirs under Consideration. (h.) Height of the water surface below the weirs, below or above the level of the crest. (h_1)								
		—0.198	0	+0.198	+0.394	+0.591	+0.787	+0.984	+1.181	
0.492	0.456	0.459	0.472	0.515	0.591	0.591	0.591	0.591	0.591	*Series No. 195.* Upper face vertical;lower face 1 to 1.
0.656	0.601	0.611	0.620	0.653	0.722	0.810	0.810	0.810	0.810	
0.820	0.748	0.752	0.765	0.807	0.869	0.942	1.024	1.024	1.024	
0.984	0.893	0.899	0.909	0.939	1.000	1.080	1.158	1.237	1.237	
1.148	1.040	1.043	1.060	1.096	1.151	1.217	1.286	1.352	1.352	
1.312	1.184	1.194	1.214	1.254	1.302	1.355	1.408	1.470	1.470	
0.492	0.485	0.485	0.489	0.512	0.575	0.575	0.575	0.575	0.575	*Series No. 196.* Upper face vertical;lower face 2 to 1.
0.656	0.643	0.643	0.646	0.666	0.702	0.797	0.797	0.797	0.797	
0.820	0.797	0.797	0.804	0.813	0.843	0.919	1.033	1.033	1.033	
0.984	0.948	0.948	0.948	0.964	0 987	1.037	1.136	1.250	1.250	
1.148	1.109	1.109	1.109	1.116	1.136	1.168	1.230	1.348	1.348	
1.312	1.267	1.267	1.267	1.267	1.282	1.312	1.365	1.437	1.437	
0.492	0.528	0.528	0.528	0.528	0.551	0.669	0.669	0.669	0.669	*Series No. 197.* Upper face vertical;lower face 5 to 1.
0.656	0.696	0.696	0.696	0.696	0.699	0.745	0.889	0.889	0.889	
0.820	0.863	0.863	0.863	0.863	0.863	0.886	0.961	1.076	1.076	
0.984	1.030	1.030	1.030	1.030	1.030	1.033	1.073	1.174	1.325	
1.148	1.201	1.201	1.201	1.201	1.201	1.201	1.217	1.279	1.388	
1.312	1.368	1.368	1.368	1.368	1.368	1.368	1.368	1.404	1.483	

Table Showing the Heads Corresponding to the same Discharge over Weirs with Crests 0.20 m. = 0.66 ft. wide, and Sloping Faces, for Various Elevations of the Surface of the Retained Water.

0.492	0.528	....	....	0.532	0.522	0.653	0.653	0.653	0.653	*Series No. 198.* Upper face inclined, 1 to 2; lower face inclined, 1 to 1.
0.656	0.682	....	....	0.686	0.694	0.745	0.889	0.889	0.889	
0.820	0.817	....	....	0.817	0.830	0.863	0.964	1.103	1.103	
0.984	0.954	....	....	0.954	0.961	0.981	1.053	1.188	1.188	
1.148	1.090	....	....	1.090	1.090	1.106	1.155	1.273	1.273	
1.312	1.217	....	....	1.217	1.267	1.230	1.267	1.371	1.371	
0.492	0.538	....	....	0.542	0.555	0.646	0.843	0.843	0.843	*Series No. 200.* Upper face inclined, 1 to 2; lower face inclined, 2 to 1.
0.656	0.686	....	....	0.689	0.696	0.732	0.889	1.063	1.063	
0.820	0.833	....	....	0.837	0.840	0.856	0.939	1.116	1.286	
0.984	0.974	....	....	0.974	0.981	0.987	1.030	1.178	1.361	
1.148	1.113	....	....	1.113	1.119	1.123	1.148	1.217	1.418	
1.312	1.257	....	....	1.257	1.260	1.267	1.276	1.315	1.457	
0.492	0.542	....	....	0.545	0.551	0.623	0.623	0.623	0.623	*Series No. 202.* Upper face inclined, 1 to 2; lower face inclined. 3 to 1.
0.656	0.699	....	....	0.702	0.706	0.732	0.850	0.850	0.850	
0.820	0.850	....	....	0.850	0.856	0.860	0.912	1.080	1.080	
0.984	1.000	....	....	1.000	1.000	1.004	1.024	1.109	1.276	
1.148	1.145	....	....	1.145	1.145	1.148	1.158	1.197	1.322	
1.312	1.289	....	....	1.289	1.289	1.289	1.295	1.319	1.375	
0.492	0.542	....	....	0.542	0.548	0.623	0.623	0.623	0.623	*Series No. 203.* Upper face inclined. 1 to 2; lower face inclined, 4 to 1.
0.656	0.706	....	....	0.706	0.709	0.722	0.843	0.843	0.843	
0.820	0.856	....	....	0.856	0.856	0.863	0.899	1.043	1.043	
0.984	1.010	....	....	1.010	1.010	1.014	1.027	1.090	1.276	
1.148	1.161	....	....	1.161	1.161	1.164	1.171	1.197	1.282	
1.312	1.312	....	....	1.312	1.312	1.312	1.319	1.332	1.375	
0.492	0.548	....	....	0.548	0.551	0.630	0.630	0.630	0.630	*Series No. 204.* Upper face inclined, 1 to 2; lower face inclined, 5 to 1.
0.656	0.706	....	....	0.706	0.725	0.833	0.833	0.833	0.833	
0.820	0.863	....	....	0.863	0.866	0.873	0.906	1.037	1.037	
0.984	1.024	....	....	1.024	1.024	1.030	1.090	1.263	1.263	
1.148	1.178	....	....	1.178	1.178	1.178	1.181	1.204	1.289	
1.312	1.332	....	....	1.332	1.332	1.332	1.335	1.345	1.385	

Mr. Nelles. From a consideration of the foregoing and other experiments on the flow over submerged weirs, the writer is impressed with the idea that the usual laboratory methods are to some extent defective and misleading, in that the conditions are artificial, and differ, except for the rare case of a flow into a reservoir, from those met in actual practice. In the laboratory experiments, the level of the backwater is regulated by impeding the flow away from the weir, and decreasing the velocity of departure; whereas, in practice, in the usual case of a dam in a flowing stream, the conditions are natural, and the height of the water surface below and the fall over the dam depend upon the quantity of water passing. It is not unlikely that some of the discrepancies hereafter noted are due to this cause.

The most common use to which engineers apply the theory and formulas for the flow over submerged dams is in connection with the slack-water improvement of rivers, for the determination of the effect of the dams in raising the water surface above them, and for the determination of the fall at the dams, in order to fix the height of the lock walls and the stage at which navigation over the dam itself will become possible.

Several formulas have been proposed for this purpose. That which is most generally used is based on the theory that the flow above the level of the lower pool is similar to a free discharge into air, and that the usual weir formulas for such discharge are applicable for its determination. The flow between the level of the lower pool and the crest of the dam is considered similar to that from a submerged orifice, and determined by the recognized formulas for such flow. By combining the common expressions for the two forms of flow, the following general formula (which should be corrected for the effect of the velocity of approach), is obtained:

$$Q = C'L\sqrt{2gz}\,(h_1 + \tfrac{2}{3}z) \quad \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

in which

Q = discharge, in cubic feet per second;
C' = a coefficient;
L = crest length of dam, in feet;
z = fall at dam, $= h - h_1$;
h_1 = height of lower pool above crest of dam.

Several methods of taking into account the effect of the velocity of approach have been advanced. The simplest of these, although not the most accurate, which will be used in the present discussion, consists of increasing the head to which the velocity of flow is due, by the head to which the mean velocity of approach is due, making the quantity under the radical in Equation (*b*)

$$\sqrt{2g\left(z + \frac{v^2}{2g}\right)}$$

The theory of Equation (*b*) makes the coefficient C' equal to 1.5 times the coefficient m of the free portion of the discharge. Mr. Nelles.

Francis, from small-scale experiments on sharp-crested vertical weirs, makes this ratio 1.38. D'Aubisson gives it at 1.43. Weisbach accepts the value assigned by theory.

The results of a large number of Bazin's experiments, Series Nos. 200 and 201, have been applied to Formula (*b*) by the writer, giving values for the coefficient C' of from 0.71 to 1.20, and of the ratio $\frac{C'}{m}$ of from 1.54 to 2.40. These results, which included the effect of the velocity of approach, showed, without apparent reason, such a wide variation that no satisfactory conclusions could be drawn from the investigation.

From a careful study and manipulation of the gauge records at the locks and dams on the Kentucky River, covering 65 days in February, March and April, 1886,* it has been possible to deduce approximate values for the ratio $\frac{C'}{m}$ for these dams. The dams consist of rock-filled cribs, with planked up-stream faces sloping at 3 to 1, and with vertical steps below the crest. The height of the dams is about 20 ft.

The deductions were made as follows: From the gauge records, periods of nearly uniform flow and conditions were selected, during which at least one of the dams was submerged, while one or more of the adjacent dams was not. Then, from the mean head for the selected period at the dams not submerged, a probable mean discharge was obtained by the usual weir formula. This discharge was assumed to be that at the dam under submerged flow, and, with the other mean conditions shown by the records to exist at the submerged dam, was substituted in Formula (*b*), which then gave the corresponding value for the coefficient C'. By comparing the value thus obtained with the value of the coefficient m, used in determining the free discharge, the ratio $\frac{C'}{m}$ was obtained.

In this way, three separate determinations were made. The first, covering four days, when the water was falling slowly, Dams Nos. 4 and 5 being under free flow, and Dam No. 3 submerged, giving $\frac{C'}{m} =$ 1.50.

The second, covering three days, while the water was falling slowly, Dams Nos. 3 and 4 being under free flow, and Dam No. 2 submerged, giving $\frac{C'}{m} = 1.53$.

The third, covering five days, during which the river rose and fell

* Annual Report, Chief of Engineers, U. S. A., 1887, pp. 1318 and 1319.

Mr. Nelles. slowly, Dams Nos. 2 and 3 being under free flow, and Dam No. 1 submerged, giving $\frac{C'}{m} = 1.46$. The mean value of the ratio $\frac{C'}{m}$ for the 11 days being 1.49.

The depths on the crest ranged from 4 to 7.5 ft., with a mean of 5.3 ft.

Another formula, used frequently for the same purposes as Formula (*b*), is known as the Chanoine and Mary Formula.

$$Q = M L h_1 \sqrt{2 g z} \qquad (c)$$

This is of the same form as the usual submerged orifice or sluice formulas, and, while it is applicable to conditions identical with those for which the coefficient M is known, it cannot be safely used otherwise. This is demonstrated by Edward Sawyer, M. Am. Soc. C. E., in an article,* in which he illustrates his assertion by assuming a case wherein the level of the upper pool remains constant, while that of the lower pool is lowered. For this case, the formula gives a decreasing value for Q, whereas the contrary would actually result.

The formula given by Chanoine and DeLagrene, in their memoir on the dams of the Upper Seine, is very generally used by French engineers. This formula is based on the theory that for a given discharge, the effect produced by a submerged dam, or other impediment to flow, in raising the level of the water surface above it, is represented by the difference between the heads to which the mean velocity at the dam site is due, before and after the execution of the works, multiplied by a coefficient to which is usually given a value of 1.5. This theory is expressed by the following formula:

$$z = K \left(\frac{V_1^2 - V^2}{2 g} \right) \qquad (d)$$

The writer has tested this formula by the substitution of the data taken from Bazin's Series No. 200. The resulting values for the coefficient K ranged from 0.25 to 1.45, and were so erratic that the selection of even a probable value was found practically impossible. A further test was made by using the results of the actual observations on the flow over the twelve dams on the Upper Seine.† In this way K was found to vary, without apparent reason, independent of z, from 0.30 to 2.50.

The most elaborate published account of the flow over submerged dams of practical dimensions and form is found in a paper by Mr. R. H. Rhind.‡ This paper describes and discusses in considerable detail observations made at a number of large masonry dams in India. Most of the dams have flat crests, with paved slopes. The exact dimensions in these particulars are not given. The discharge was calculated by

* *Van Nostrand's Magazine*, Vol. xxxiv, p. 176.

† *Annales des Ponts et Chaussées*, 1868.

‡ *Minutes of Proceedings*, Institute of Civil Engineers, for 1886.

Mr. Nelles.

TABLE No. 22.—DIMENSIONS OF AND OBSERVED CONDITIONS OF FLOW AT CERTAIN LARGE MASONRY DAMS IN INDIA, WITH CORRESPONDING COEFFICIENTS OF DISCHARGE.*

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
		MAKE UP OF WEIR.		DEPTH OF CREST BELOW WATER SURFACE IN							COEFFICIENTS OF DISCHARGE IN THE VARIOUS SUBMERGED-FLOW FORMULAS.					
												Formula b, observed heads.		Formula a, observed heads.		
Name of river.	Name of weir.	Description of parts.	Length of parts, in feet.	Upper pool, feet.	Lower pool, feet.	Observed fall over weir, feet.	Mean surface velocity of approach, in feet per second.	Corresponding velocity head, feet.	Calculated discharge, in cubic feet per second.	Height of main body of weir, feet.	Formula e, observed head corrected for velocity of approach.	Corrected for velocity of approach.	Not corrected for velocity of approach.	Corrected for velocity of approach.	Not corrected for velocity of approach.	Approximate section of weir.
			L	h	h_1	$h - h_1 = z$	v	$\frac{v^2}{2g}$	Q	p						
		Main body	665	19.21	18.35											
Brahmini	Pattia	First step	60	8.21	7.35	0.86	7.84	0.96	114 000	...	0.881	0.806	1.172	0.241	0.248	
		Second step	9	2.21	1.35											
		Sluices	100	18.10	15.29											
		Piers	14	7.68	4.87	2.81			Total							
	Byturnee	Main body	912	10.84	8.03				260 000	7.9	0.953	0.940	1.125	0.491	0.502	
Byturnee		Total overfall	1 026	......	......	....	8.51	1.13								
		Sluices	100	18.60	15.20				For main							
	Burrah	Piers	14	8.18	4.78	3.40			body alone	7.9	0.979	0.970	1.148	0.511	0.534	
		Main body	412	11.34	7.94				212 400							
		Total overfall	526													
		Sluices	389	18.02	17.55											
		Piers	140	8.02	7.55	0.47	6.99	0.76	....	7.5						
Brahmini	Brahmini	Main body	3 471	10.52	10.05	....	......	....	260 000		0.878	0.790	1.250	0.266	0.257	
		Total overfall	4 000	......	......	....	......	....	307 500		0.876	0.786	1.275	0.245	0.252	
		Sluices	90	30.675	30.15											
Manhanuddy	Kajooree	Breaches	324	25.615	25.09	0.525	11.62	2.10	....	10.65	......	......	......	......	......	
		Main body	3 433	20.115	19.59	....	......	....	680 000		0.893	0.764	1.710	0.261	0.274	Same as above.
		Total overfall	3 847	......	......	....	......	....	787 000		0.891	0.763	1.705	0.355	0.267	
		Shore sluices	190	17.45	15.45											
Manhanuddy	Manhanuddy	Center sluices	460	17.20	15.20	2.00	7.74	0.93	....	6.00	......	......	......	......	......	
		Main body	5 696	11.45	9.45	....	......	....	488 000		0.595	0.561	0.679	0.265	0.276	Same as above.
		Total overfall	6 346	......	......	....	......	....	609 000		0.633	0.597	0.722	0.274	0.284	
		Sluices	255	13.30	12.10											
Manhanuddy	Beropa	Piers	118	4.30	3.10	1.20	6.65	0.68	....	5.00	......	......	......	......	......	
		Main body	1 607	8.30	7.10	....	......	....	82 800		0.594	0.570	0.715	0.244	0.255	Same as above.
		Total overfall	1 980	......	......	....	......	....	112 700		0.648	0.677	0.846	0.260	0.271	

* Compiled and calculated from Paper No. 2171. *Minutes of Proceedings*, Institution of Civil Engineers. Vol. LXXXV, p. 307.

Mr. Nelles. the Humphrey and Abbott Formula, and covers an unusually wide range. Table No. 22, which has been prepared from Mr. Rhind's paper, shows the shape and known dimensions of the dams, the calculated discharge, the observed conditions and deduced coefficients.

In deducing his coefficients, Mr. Rhind used the formula

$$Q = C_1 \, L \sqrt{2\,g} \left[\, h_1 \sqrt{z + 0.01\,v^2} + \frac{2}{3}\, z \sqrt{z + 0.035\,v^2} \right] \ldots\ldots\ldots\ldots (e)$$

in which v represents the mean surface velocity of approach. This formula may be reduced to the same form as (b) by neglecting the correction for the velocity of approach.

In addition to the information and results taken from Mr. Rhind's paper, the writer has calculated from the same data and shown in Columns 13 to 16 of Table No. 22, the coefficients of discharge for Formulas (a) and (b); first, considering the effect of the velocity of approach, by increasing the observed head an amount equal to $\frac{v^2}{2\,g}$; second, not considering the effect of the velocity of approach, and using the observed heads in the formulas.

The results in Table No. 22 indicate clearly the necessity for specially considering the effect of the velocity of approach in connection with Formula (b), and show a marked uniformity in the value of the coefficient m in Formula (a), notwithstanding a very wide and inexplicable variation in the values of the coefficient C'.

Van Nostrand's Magazine, Vol. xxxii, page 473, gives the following details of certain observations at the Bazacle Dam in the Garonne River at Toulouse:

$h = 6$ m.;
$h_1 = 5$ m.;
$z = h - h_1 = 1$ m.;
$L = 283$ m.;
$Q = 6500$ cu. m. per second.

Substituting these quantities in Formula (b), we find $C' = 0.915$. Correcting z for a mean velocity of approach of 3.4 m. per second, we find $C' = 0.724$. In the same way, by means of Formula (a), we find $m = 0.353$ and 0.336.

This discussion has been carried to a considerable length in order to illustrate the present state of our practical knowledge on the general question of flow of water over dams, and, aside from the Bazin experiments, much of the matter presented has very little intrinsic value except for this purpose. It is hoped, however, that the illustrations given may attract the attention of skilled investigators to this field, and thus indirectly lead to a more satisfactory solution of the problem of submerged flow.

Some valuable and interesting features of the Bazin experiments

have not been noticed by Mr. Rafter; principal among these are the distribution of pressure and velocity in the nappe. Concerning these features, Bazin says in his summary: Mr. Nelles.

"The pressure under the nappe has been measured in nearly every experiment. For a certain number of them we have determined the interior distribution of both the pressure and velocity. This distribution presents a theoretic interest, but cannot, like the pressure under the nappe, be put to practical use. For a weir of given form, the coefficient m depends principally on two elements, the contraction at the sill and the pressure under the nappe. This pressure is so clearly related to the discharge that we can, by proper observations, deduce from it the variation in the discharge, with a precision not to be obtained by the direct measure of the heads."

George W. Rafter, M. Am. Soc. C. E. (by letter).—Mr. Williams is, in general terms, right in his statement that at the beginning of the experiments it was agreed that the reduction of the experiments should be according to Bazin's formula; but so much more time was taken than was expected, that when the writer came to the reductions, and several difficulties developing, due to the experiments going so far beyond the limits of Bazin's formula, it was found necessary to adopt a modified form which would permit of more rapid work. Mr. Rafter.

Mr. Williams has, however, used the Bazin formula rigidly, with the result of showing differences in extreme cases of 3 per cent.

As stated in the paper, Bazin's formula includes the velocity of approach quite as much as any other formula. The reduction of the experiments on the basis of neglecting this element is, for high heads, therefore, an error, and may, with certain others, be briefly discussed. The more especially is this true since it is impossible to work on the ratio $\frac{m}{m'}$.

The following are the more important points, then, in which it appears to the writer that Mr. Williams is in error:

(1) As to the statement that Bazin's formula necessarily provides for the effect of velocity of approach in its coefficient: After determining his two equations, as follows:

$$Q = n\left(1 + \frac{3}{2}\, a\, \frac{u^2}{2gh}\right) l h\sqrt{2gh} \ldots\ldots\ldots\ldots \quad (1)$$

and

$$Q = n\left[1 + K\, \left(\frac{h}{p+h}\right)^2\right] l h \sqrt{2gh} \ldots\ldots\ldots \quad (2)$$

Bazin says:

"The first of our two formulas may be found preferable in certain theoretical researches; but the second is obviously more convenient in practice, since it does not contain the velocity of approach u, which itself depends upon the discharge, but only the directly measurable elements, h and p. We, therefore, give it the preference."

Again, on page 258, Bazin says:

"Instead of considering on the other weirs the absolute values of

Mr. Rafter. the coefficient m, we have compared them with the coefficient m' for a free nappe, for the same head on a sharp-crested weir of the same height. This substitution of the ratio $\frac{m}{m'}$ for the absolute values of m eliminates, in a large measure, at least, the influence of velocity of approach, and facilitates greatly the discussion of results."

It appears, therefore, that Bazin did not include the velocity of approach in his formula, and hence its effect is to be allowed for, the same as in the other formulas.

(2) Mr. Williams' view, that the correction for velocity of approach is subtracted, is novel, and will so strike most hydraulicians. It is tolerably clear that he has not understood the language, for certainly the writer has not proceeded in the way that Mr. Williams has assumed. Just how this correction is made will be clear on reference to the correction sheets appended.

(3) It is not believed that the writer has misunderstood Fteley and Stearns. What they actually stated is as follows:

"The head, if measured outside of the angle of pressure, should be taken far enough up-stream from the weir to represent the height of the water surface above the beginning of the surface curvature, *i. e.*, at a distance from the weir equal to $2\frac{1}{2}$ times its height above the bottom of the channel. So great a distance from the weir may not be necessary with deep channels; little harm, however, can result from taking the head too far from the weir if the channel is uniform; the amount of error being only the loss of head due to friction."

It would appear, therefore, that the condensed quotation of Fteley and Stearns, namely: "that the only inaccuracy to come from measuring the heads more than 6 ft. back will be due to surface slope," is strictly true.

(4) Mr. Williams' discovery that it is possible to use such a form of weir for a Venturi meter is rather ancient. If he will turn to the *Annales des Ponts et Chaussées*, for 1898, he will find that Bazin discusses this matter very extensively, and concludes in the following words:

"The measure of pressure can then be utilized for registering varying flows. An instrument founded upon an application of this principle has been used for a number of years in America for measuring the flow of large water conduits. It is understood, from a well-known experiment of Venturi, that if we restrict for a short distance the diameter of a conduit, it will produce, in the part restricted, a diminution of pressure corresponding to the increase in velocity of the fluid, etc. * * * The apparatus designed by Clemens Herschel is founded on this principle. The same principle is evidently applicable to weirs where one can realize directly the registration of the discharge by employing the process of Mr. Hegly for transforming the indication of height on a weir into indications of flow."

(5) Mr. Williams details an experiment as follows:

"At the time that these heads were read a tape was tacked upon the wall of the canal vertically at the crest, so that the top of the sheet could be read thereon at the same time that the pressure in the piezome-

ter along the crest of the weir was read. With a head up stream of 97 cm. the tape read 70 cm. and the piezometer at the crest read 38 cm." Mr. Rafter.

Mr. Williams then adds: "There is the effect of velocity upon the head." Exactly what Mr. Williams means by the effect of velocity upon the head, the writer does not know, but the obvious interpretation of the experiment is that there is a suction at the crest, as pointed out by Bazin some time ago, and that the piezometer reading 38 cm. merely recorded such suction.

(6) After somewhat careful study of the work of Francis, of Fteley and Stearns, and of Bazin, the writer is obliged to differ from Mr. Williams in regard to the accuracy with which the respective experimenters did their work. He has no doubt that Bazin's work was quite as accurate as that of the others. Mr. Williams, moreover, raises a question of patriotism which, it seems to the writer, would be better left unsaid.

(7) We come now to the question of the proper location of the piezometers. As detailed in the paper, two sets of piezometers were used. In one case a 1-in. galvanized iron pipe, with holes ¼ in. in diameter and spaced 6 ins. apart, was laid across the channel about 8 ins. above the bottom, with the holes therein opening downward. Connections with these pipes were made by ¾-in. pipes passing through the bulkhead to a point below the weir, where the gauges could easily be connected by rubber hose. In order to check the accuracy of the piezometric readings, at the conclusion of Experiment No. 17, a fourth piezometer pipe was set in the bottom of the flume above the lower bulkhead, and about 6 ins. up stream from the upper piezometer. This pipe was set with ¼-in. holes directly on top, and with the top of the pipe flush with the bottom of the flume. As stated, considerable differences were noted in simultaneous readings of these two piezometers.

Mr. Williams describes the piezometer which he finally adopted as raised 6 ft. from the bottom, and he apparently asks that measurements taken thereon shall be considered more accurate than when taken on a piezometer flush with the bottom As a reason for adopting this type of piezometer, he assigns that Mr. FitzGerald used it in his experiments on the flow of water in a 48-in. pipe (except that Mr. FitzGerald's piezometer was on the bottom), although it seems clear enough that the difference of condition would indicate no comparison between Mr. FitzGerald's method and the work at Cornell. It is perfectly true that a memorandum sufficient to locate a correction curve was received from Mr. Williams and rejected, for the reason that his views on piezometers, particularly that a piezometer 6 ft. from the bottom was preferable to one flush with the bottom, were not considered especially sound. For this, the writer is fully responsible. It is not intended to state, however, that the flush piezometer was absolutely the best that

Mr. Rafter. could be applied, but it seems to be clear enough that it was better than one 6 ft. up, where the current necessarily must have been considerable.

It seems difficult to understand why there should be so much uncertainty about piezometers, the more especially as Mr. Hiram F. Mills has pointed out in his "Experiments upon Piezometers used in Hydraulic Investigations"* the proper method of their use. In his conclusions Mr. Mills states:

"This result indicates, with a nearness of approximation unusual in hydraulic investigations, that with the plane of the orifice accurately in the plane of the side of the conduit, the piezometer will indicate the true height of the surface of the stream."

Without going into detail, it may be remarked that the writer believes that, in order to obtain accurate results, all Mr. Williams had to do was to place the plane of the orifice accurately in the plane of the side of the conduit. Evidently, from the experiments detailed, he finally reached this result; thus experimentally realizing a method which has been the common property of engineers for many years.

It is clear, also, that he has considerable doubt as to the accuracy of his method of measurement, because he finally states that "it cannot be affirmed that the new piezometer is a correct one to use with Bazin's formula."

(8) Mr. Williams' argument as to the method of reduction having been shown to be wrong, it follows that his tables have no special significance, and his expression of opinion, that the results do not come nearer to accuracy than 6%, even though on his own unfavorable assumption he is unable to find more than 3% error, is, it seems to the writer, going somewhat farther than is necessary.

(9) Mr. Williams criticises the brevity with which the results are presented, his view being that the experiments should be presented with as little reduction as possible, "so that in the future the investigator may determine for himself, in the light of such new knowledge as he may then have, just what reliability is to be put upon the observations, and what lessons are to be drawn from them."

This view is based upon the notion that all the work done by engineers is highly scientific. As a matter of fact, the engineer is not specially a man of science, but, nevertheless, he should obtain accurate results. The method of writing in such detail as Mr. Williams proposes has always seemed to the writer somewhat sophomoric. He has the detail, however, and takes pleasure in presenting it, although the paper has already grown to undue length.

(10) In Figs. 21 and 22, Mr. Williams presents a series of sketches of the curves formed by the water as it flows over the various weirs. Inasmuch as Bazin has presented similar sketches, but so much more accurate than his, it is difficult to see the utility of this presentation.

* *Proceedings*, American Academy of Arts and Sciences, 1878.

(11) Mr. Williams omits Series Nos. 1, 2 and 6, and also considers Series No. 5 as quite possibly inaccurate. The writer considered Series Nos. 1, 2, 3 and 4 as less accurate than the others. In regard to these, it may be remarked that the piezometric readings were not as accurate as might be desired. Fortunately, the actual elevations have been kept, and have been used in the reductions. The actual elevations, however, were not measured closer than about $\frac{1}{100}$ ft., and within that limit there may be some variation. Otherwise, these four experiments are as reliable as the others. Mr. Rafter.

(12) Mr. Williams states that:

"One of the most important facts brought out in the past year's investigations in the Cornell Hydraulic Laboratory has been the formation of a vacuum more or less perfect behind the falling sheet when air is not freely admitted."

Why Mr. Williams should claim this as an original discovery is difficult to determine, the more especially since Bazin has discussed it very extensively.

(13) Mr. Williams' erroneous view as to the proper method of computation by Bazin's formula has a number of important considerations connected therewith. For instance, he is apparently under the impression that the influence of the height of the weir is a matter of so much importance as to require computing to the second or third decimal place. The following demonstration, for which the writer is indebted to Alfred Noble, M. Am. Soc. C. E., may serve to set him right on this question: Taking Bazin's formula—

$$m = n\left[1 + 0.55\left(\frac{h}{h+p}\right)^2\right]$$

we can deduce

$$\frac{n}{m} = \frac{1}{1 + 0.55\left(\frac{h}{h+p}\right)^2}$$

We can then calculate values of $\frac{n}{m}$ for assumed values of $\left(\frac{h}{h+p}\right)$, obtaining the following table:

$\left(\frac{h}{h+p}\right)$	$\left(\frac{n}{m}\right)$
0.1	0.995
0.2	0.978
0.3	0.953
0.4	0.919
0.5	0.879
0.6	0.835
0.7	0.788
0.8	0.740
0.9	0.692
1.0	0.645

Mr. Rafter. This formula is empirical, is not applicable to extreme cases, and is obviously absurd for values of $\left(\frac{h}{h+p}\right)$ approaching unity.

(14) The Croton gaugings have been referred to, and Mr. Williams expresses the opinion that they are considerably more accurate than those now under discussion. On this point the writer desires to state that without making a very thorough study of the Croton gaugings, he has computed the values of m and plotted coefficient curves similar to those on pages 267–284. These curves are not especially reassuring as to the minute accuracy of the results. Some of them are undoubtedly in error.

Again, consulting the detailed tabulation, as published in Mr. Freeman's Report on the New York Water Supply, pages 137–141, it is learned that, out of 139 experiments, 44 were omitted because of uncertainty of some sort in the detail. This is nearly 32 per cent. As indicating a probable reason why these experiments are somewhat unreliable, it may be noted that they were all made with a longitudinal piezometer, raised 6 ft. from the bottom; which position the writer believes is fundamentally wrong. Mr. Freeman will understand, it is hoped, that the slight criticism of the Croton experiments made here is not in any way directed toward him. It is directed alone to Mr. Williams' statement that the Croton experiments are considerably less than 6% in error.

(15) Mr. Williams has referred to a Francis piezometer. This also seems to be an error, as the writer does not now remember that Francis used anything that could with propriety be called a piezometer. In Francis' original experiments, the heads were measured in "still boxes" which were set in the hollow quoins of the canal locks. These "still boxes" had a hole in the bottom, and were set in practically still water, the level of the water being determined by a hook gauge. Later, other experimenters set the "still boxes" at any convenient place and connected them with the water above the weir by means of pipes.

The piezometer described on page 322 is the Fteley and Stearns piezometer.

The writer very readily admits that the several formulas are not strictly comparable, and has therefore revised the Bazin experiments to conform therewith. This revision involves the dropping of the C from $C = m\sqrt{2g}$, retaining only $m\sqrt{2g}$. In the case of the Cornell experiments, however, owing to the method of reduction being different, the expression $C = M\sqrt{2g}$ is perfectly true, and has accordingly been retained. Practically, the difference is so small as to be, in the great majority of cases, safely neglected; but, in order to keep the distinction between the theoretically true and practically true distinct, the change is made. Still, for all ordinary cases, the Bazin coefficients may be used in the expression $C = m\sqrt{2g}$, thus very greatly extending the range of weir formulas.

In his pamphlet on "Measuring Water," Clemens Herschel, M. Am. Soc. C. E., has discussed nearly every phase of the question, and defined with great clearness the more important elements of the problem. This pamphlet may, in the writer's opinion, be profitably read by anybody having occasion to use the weir. Mr. Rafter.

(16) Mention has been made, at the beginning, of special difficulties of reduction. These relate more especially to the mean values of a and K. For instance, within the limits of Bazin's experiments, a ranges in value from 2.43 to 1.40, and K from 0.77 to 0.50. These ranges in value occur for weirs from 2.46 ft. (0.75 m.) in height to 0.79 ft. (0.24 m.) in height. The standard weir of the Cornell experiments was 13 ft. in height and the experimental weir about 5 ft. The values used are means derived from a considerable number of cases. These are given in tables.*

These tables show:

(1) That the values of a decrease with decrease of height of weir, and that, for a given height, they first increase with the head, up to a maximum, and then again decrease.

(2) That the values of K, which vary less than those of a with the height of the weir, become more nearly equal as the head increases.

There seems to be a general misconception as to Bazin's use of the correction for velocity of approach. In regard to this element, Bazin states:

"When it is required to take into account the effect of the velocity of approach to the weir, it is usual, in the very simple formula,

$$Q = m\, l\, h \sqrt{2 g h} \quad \ldots\ldots (1)$$

to substitute $\left(h + a \frac{u^2}{2 g}\right)$ for h, where u is the mean velocity in the channel leading to the weir and a a coefficient not very accurately determined, but generally taken at 1.5. The equation thus becomes:

$$Q = n\, l \left(h + a \frac{u^2}{2 g}\right) \sqrt{2 g \left(h + a \frac{u^2}{2 g}\right)}$$

$$= n\, l\, h \sqrt{2 g h} \left(1 + a \frac{u^2}{2 g h}\right)^{\frac{3}{2}} \text{"} \quad \ldots\ldots (2)$$

After a series of transformations, this reduces to

$$Q = n \left(1 + \frac{3}{2}\, a \frac{u^2}{2 g h}\right) l\, h \sqrt{2 g h} \quad \ldots\ldots (3)$$

an expression which may be written in nearly equivalent form,

$$Q = n \left[1 + K \left(\frac{h}{h + p}\right)^2\right] l\, h \sqrt{2 g h} \quad \ldots\ldots (4)$$

which does not contain the velocity of approach, u, but only the directly measurable elements, h and p.

* See *Annales des Ponts et Chaussées* for 1888; and *Proceedings*, Engineers' Club of Philadelphia, Vol. VII, No. 5. January. 1890.

Mr. Rafter. We, therefore, Bazin states, adopt definitely for m the formula which gives, for the discharge Q,

$$Q = n \left[1 + 0.55 \left(\frac{h}{p + h}\right)^2\right] l\, h \sqrt{2 g h},$$

and for value of M,

$$M = n \left[1 + 0.55 \left(\frac{H}{p + H}\right)^2\right].$$

It is hoped it will be understood that this is a very brief exposition of these formulas. Several pages are condensed into one. So far as the writer is concerned, he had no intention of writing a treatise on the weir.*

The writer is aware that a slight uncertainty attaches to the position of the piezometer, as noted by Mr. Parmley, nor would he be satisfied with readings to 2 mm. in measuring low heads. It seems, however, that Mr. Parmley's criticism is, on the whole, pointless, precisely because the heads are large. Indeed, his demonstration is pertinent on this point.

As to the various criticisms relative to the form of the coefficient curves, the writer wishes to state that, on the whole, they seem to indicate a lack of familiarity with the subject, which, indeed, is not to be wondered at, considering how thoroughly engineers have been wedded to the view that Francis' formula was all there was to the weir. Apparently, the insignificant causes which may produce a very material variation in the form of the curve are not appreciated. Moreover, a very considerable variation in the form of the curve may be produced by varying the value of the coefficient only very slightly, as fairly illustrated by comparing the two curves of Experiment No. 6. The writer cannot but think, therefore, that Mr. Wisner has been somewhat too hasty in his conclusion that some of the curves are wrong. As the matter stands, there is not, in the writer's opinion, any basis for the judgment that the curve for Experiment No. 19 is wrong. As to the observations made since these experiments were completed, and which show material error in these observations, it may be merely pointed out that as yet they have not materialized.

A portion of Mr. Parmley's discussion is conflicting. To begin with, he states (page 349):

"If the writer understands Bazin's formula, no correction for velocity of approach is necessary, since that is taken account of already in the formula and the coefficients given in his table."

On page 350, he states:

"One of the first things noticed was the fact that in correcting for velocity of approach, Bazin followed practically the method of Francis, but with a coefficient which gives a larger correction."

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* In order to obtain the whole matter conveniently, the reader is referred to the translation by Marichal and Trautwine, *Proceedings*, Engineers' Club of Philadelphia Vol. VII, No. 5, January, 1890.

Mr. Rafter.

A word as to the standard weir, 16 ft. in length: Mr. Kuichling has expressed a doubt as to its accuracy, and cites Hamilton Smith's remark, "such speculations, though possibly ingenious and plausible, when tested with facts, generally prove to be very wide of the truth." Mr. Kuichling might, with great appropriateness, have quoted the preceding sentence. As he failed to do so, probably from mere inadvertence, the writer will quote it: "It is hardly worth while to pursue this subject still further into spaces where we have absolutely lost the guidance of experimental light." Does Mr. Kuichling mean to affirm that we have "absolutely lost" the guidance of experimental light in these experiments? As to his assumed error in computation, on looking over the detail of the experiments he may conclude that possibly he is in error in hastily assuming error in computation.

Probably the most appreciative discussion is that of Mr. Nelles, who evidently has had occasion to study the subject broadly. Mr. Horton's discussion is also one from a man who has had to deal with large weirs. The writer congratulates himself on having called Mr. Nelles' attention to Bazin's work, and hopes that the study so well begun may be continued to a successful issue. Mr. Nelles thinks, however, that there may be a question either as to the accuracy of these experiments or their applicability to the flow over structures differing in shape and dimensions from the experimental forms. On this point the writer again calls attention to what seems to him ought to be understood by everybody, namely, that very slight change in the form of the weir often produces relatively important changes in the form of the coefficient curve. Reasoning from one coefficient curve to another is, therefore, without the slightest significance. This point is again referred to because of a general misapprehension by those who have contributed to this discussion.

Mr. Kuichling gives an account of the foundation of Bazin's formula, although the writer cannot but think it would have been much more acceptable if he had included some account of how Bazin corrects for velocity of approach. Lacking such, it indicates rather casual reading of Bazin's paper.

In the writer's opinion, Mr. Williams is wrong in his view that Bazin intended to apply his formula for height of weir rigorously. In Table No. 2,* for a height of 6.56 ft. above the bottom of the channel, the coefficient *m* does not vary by so much as one decimal place between the limits of heads of 0.30 and 0.60 m. The preceding discussion may serve to indicate why this correction is a somewhat general one, and not intended to be rigorously applied beyond the limit.

With reference to the method used in calibrating the standard weir, the writer is unable to see any difficulty about it, but inasmuch as there is clearly some misapprehension he will endeavor to clear away

* See, also, *Annales des Ponts et Chaussées*, for 1888, p. 446.

Mr. Rafter. the doubt which has arisen in the minds of some as to just what was done.

The reasons why the flow in the experimental canal was not checked have already been brought up in the discussion. The writer's contention is that this was not specially necessary; that all that had to be done was to absolutely duplicate Bazin's experiments; and, up to a height of 2 ft. on the 16 ft. weir, at any rate, the error could not have been more than 1 per cent. As to the length of the weirs affecting the results, as suggested by Mr. Kuichling, Bazin's decisive experiments on his 1-m. and 2-m. weirs ought to be conclusive. He states distinctly that he was unable to note any difference in the results. It follows, therefore, that if there was no difference up to 2 m., there is no reason for supposing any difference up to 4 or 5 m. Owing to the fact that the arrangements were not absolutely duplicated, the writer thinks it quite probable that the error may range from 1 to 2 per cent. Above 2 ft., there is, of course, more question. Indeed, the writer realized that Bazin's formula, from its peculiar composition, as already pointed out in the discussion, might not be entirely reliable beyond the limit of his table.*

Column 3, of Table No. 23, gives values of n.

TABLE No. 23.

Head, in meters. h.	Head, in feet. h	n	$\frac{h}{p+h}$	$\left(\frac{h}{p+h}\right)^2$	$1+0.55\left(\frac{h}{p+h}\right)^2$	M.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.10.........	0.3281	0.4322	0.0244	0.000595	1.000327	0.4323
0.20.........	0.6252	0.4215	0.0476	0.002266	1.001246	0.4222
0.30.........	0.9843	0.4147	0.0698	0.004872	1.002680	0.4185
0.40.........	1.3124	0.4144	0.0909	0.008263	1.004544	0.4163
0.50.........	1.6405	0.4118	0.1111	0.012343	1.006789	0.4146
0.60.........	1.9686	0.4092	0.1305	0.01703	1.009366	0.4133

A discharge curve was computed for the 16-ft. standard weir, using the values of n given in Table No. 23, and from this curve the discharge over the standard weir was taken out for such observed heads in Experiments Nos. 20 and 21, as did not exceed 1.969 ft. or 0.6 m., which is the limit of the table, and it was found that the flow over the 16-ft. standard weir, under a head of 1.969 ft. produced a head of 3.57 ft. on the 6.58-ft. sharp-crested experimental weir. We have thus obtained a series of heads on, and flows over, the 6.58-ft. weir, up to 3.57 ft. head. From this point, Bazin's formula was no longer used.

The velocity of approach in the channel leading to the experimental weir was next determined directly, which was also done in all experiments

* *Annales des Ponts et Chaussées*, for 1888, p. 446 (also given as Table No. 2 of this paper).

on irregular sections, by dividing the volume of flow already known, by the actual cross-sectional area of the channel of approach. The corresponding velocity heads, so obtained, were added to the observed heads on the experimental weir, and the discharge coefficient computed by the formula, Mr. Rafter.

$$M = \frac{Q}{L\,H\sqrt{2\,g\,H}}$$

We have obtained by the preceding operations, a series of discharge coefficients for the sharp-crested weir, 6.58 ft. in length, for heads up to 3.57 ft. A discharge curve was then computed therefrom for the 16-ft. sharp-crested weir for heads up to 3 ft. on that weir, as given in Fig. 4. That is to say, the operation was, in effect, reversed. We had first obtained, with heads of 1.969 ft. on the 16-ft. standard weir, a corresponding head of 3.57 ft. on the 6.58-ft. experimental weir, and their coefficients are considered accurate to within about 1 per cent. A discharge curve was then plotted from which readings from the 6.58-ft. experimental weir were applied, back to about 3 ft. on the 16-ft. standard weir, thus calibrating that weir up to 3 ft., and consequently making it possible to obtain readings up to 5.89 ft. on the 6.58-ft. experimental weir. This explanation, it is hoped, will clear up any uncertainties as to just why the 6.58-ft. weir was used.

In obtaining the coefficients used in plotting the foregoing curve, the velocity head was added to the observed head, and hence the velocity head at the standard weir should be, and has been, added to the observed head in reducing the other experiments; but as the velocity head is a function of the discharge, it must, in this case, be determined by successive approximations. This was done in a manner similar to that already explained, and gives a continuous discharge curve for heads within the limit of the experiments, or say from about 0.5 ft. to 5 ft.

These coefficients are in a form which makes them applicable, without modification, to a weir of any height whatever. The effect of the height of the weir on the velocity of approach is therefore included in the velocity head. Mr. Williams is mistaken in the statement that they give a discharge curve for the 16-ft. standard weir agreeing with that by Bazin's formula, while that for the lower 6.58-ft. weir does not do so. It remains, therefore, only to point out that we know nothing of the flow over the experimental weirs except as determined from known flows over the standard weir. It is a mere waste of time to attempt to apply formulas for flow to the experimental weirs, as it is, also, to attempt to reason from known forms which have been experimented upon, to unknown forms. However unsatisfactory it may appear to those who are looking for simplicity, the present tendency in hydraulics is to great complexity.

The writer is aware that the point may with propriety be discussed as to whether or not absolutely the best correction for velocity of

Mr. Rafter. approach was applied in this case. All that he can state is that the pressure for immediate results was very great, and that for the time available the best was done.

For reasons given on page 301, the writer did not feel justified in fixing the percentage of probable error. Recently, however, a careful study has been made of all the data and circumstances attending the making of the experiments, and the writer, in consequence, is of the opinion that at the limit of the highest heads the error may possibly be somewhat over 2 per cent. Even though it were 3%, as Mr. Williams claims, he should not feel specially dissatisfied, because the difficulties in making an absolutely correct measurement are enormous. In any case, the writer hopes that a possible error of 3% in the determinations will not be fatal to the experiments. Up to 3.57 ft. on the experimental 6.58-ft. weir, it is probable that the error does not much exceed 1 per cent. If, therefore, Mr. Williams will kindly recompute, with proper allowance for velocity of approach at both weirs, it is believed that he will find the error to be smaller than he has imagined.

Another point in which this work differs from Bazin's may be mentioned: Bazin worked on weirs of substantially the same length, and could thus avail himself of the relation $\frac{m}{m'}$ for neutralizing the effect of velocity of approach. In the present case the standard weir was 16 ft. in length and the experimental weir was 6.58 ft. in length. It was impossible, therefore, to proceed in the same manner as Bazin.

A word as to the future of weir measurements: At the present time there is certainly a good deal that is unsatisfactory about them. Bazin's formula is undoubtedly the best that has been thus far devised, but is it, after all, the best that can be devised? Bazin has evidently realized the unsatisfactory nature of weir measurements as now conducted, for he expresses himself strongly that preferably such measurements should be made by means of pressure observations in the nappe.* From such observations it is possible there would come a decrease in the number of coefficients. The writer regrets that he is not able to offer any suggestions as to the best method of proceeding with a series of nappe observations. Mr. Williams has here an opportunity to employ himself usefully for several years.

Time will not be taken to point out the many practical relations embodied in the new views as to weirs. One deduction may, however, be pointed out, namely, the effect as to gaugings of the run-off of rivers. Wherever there were convenient dams, thus far, engineers have used Francis' formula almost exclusively for such work, although it is clear enough now that, without understanding the great variation on weirs of different forms, this formula, with its fixed coefficient, is in many cases in no way applicable, the variations being from 20 to

* See, also, an article by Clemens Herschel, M. Am. Soc. C. E., in *Engineering News*, November 10th, 1898.

40 per cent. Recently, there has been considerable discussion as to the effect of forests on run-off, but inasmuch as such effect is considerably within the limit of variation in Francis' formula, it is obvious that gaugings based thereon are too uncertain to furnish any light whatever on this subject. As far as the writer can now see, it will be necessary to recompute most of the gaugings, before this question can be settled. Mr. Rafter.

Possibly due to a slight confusion in the use of H and h, there has been some misapprehension as to just how the Cornell coefficients are to be used. At the risk of being somewhat elementary, the writer will give an illustration: In the formula, $Q = C\,L\,H^{\frac{3}{2}}$, the coefficient, C, varies according to the head instead of remaining constantly equal to 3.33, as in Francis' expression. Thus, if it is required to compute the flow over a dam of the form given by the curve on page 274 (Cornell Experiment, No. 9) for $H = 0.5$ ft., $C = 3.300$. When $H = 1$ ft., C will $= 3.570$, and when $H = 1.5$ ft., C will $= 3.595$, and so on. The values of H, it may be repeated, are corrected for velocity of approach, and are to be used without further correction of any kind.

The writer confesses to a feeling of disappointment at the results of the discussion. He had hoped that the numerous practical questions which are raised would receive extensive consideration, but, unfortunately, the discussion has been almost entirely about relatively unimportant theoretical questions. This, the writer, although in no way responsible for, very much regrets.

Enough of the tabulations of the detailed results are herewith presented to show clearly the method of reduction. It is considered that they are sufficiently self-explanatory to be readily understood by anybody. They were not given in the original paper because of its great length.

TABLE No. 24.—Experiment No. 1.—Bazin's Series No. 130.

Computed from Heads Reduced to Gauge Board at Experimental Weir.

Period.	Observed head, upper piezometer, standard weir.	Correction for middle piezometer.	Corrected head, in centimeters.	Corrected head, in feet.	Correction for velocity of approach.	Final head on standard weir, in feet.	Flow over standard weir, in cubic feet per second.	Flow over experimental weir. Corrected for leakage.	Flow over experimental weir, per foot of crest.	Observed head, experimental weir.	Mean correction, to reduce to gauge-board reading.	Corrected head, in feet.	Final corrected head on experimental weir.	Q per foot.	$C = M\sqrt{2g}$.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1...	85.59	2.94	88.53	2.905	0.016	2.920	261.7	261.21	39.73	5.494	0.522	4.972	4.972	39.73	3.584
2...	85.12	2.92	88.04	2.889	0.016	2.904	260.0	259.51	39.44	5.394	0.522	4.872	4.872	39.44	3.668
3...	85.00	2.91	87.91	2.884	0.016	2.900	259.2	258.71	39.31	5.375	0.522	4.853	4.853	39.31	3.940
4...	73.19	2.32	75.51	2.477	0.011	2.488	207.5	207.05	31.47	4.492	0.354	4.138	4.138	31.47	3.739
5...	59.41	1.68	61.09	2.004	0.006	2.010	150.1	149.70	22.81	3.573	0.205	3.368	3.368	22.81	3.690
6...	31.37	0.59	31.96	1.049	0.001	1.049	57.7	57.40	8.71	1.755	0.030	1.725	1.725	8.71	3.844
7...	21.49	0.29	21.78	0.715	0.0008	0.715	32.4	32.13	4.88	1.190	......	1.190	1.190	4.88	3.759

Mr. Rafter.

TABLE No. 25.—EXPERIMENT No. 5. MAY 27TH, 1899. A. M.

Period.	Observed head on middle piezometer, standard weir, in feet.	Correction for velocity of approach, in feet.	Corrected head, in feet.	Flow over standard weir, in cubic feet.	Flow over experimental weir, in cubic feet. Corrected for leakage.	Observed head on experimental weir, in centimeters.	Correction for flush piezometer, in centimeters.	Corrected head, in centimeters.	Corrected head, in feet.	$p + h$.	V_o.	$\frac{V_o^2}{2g}$	Final corrected head, in feet.	Q per foot.	$C = M\sqrt{2g}$.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1..	2.956	0.0169	2.973	268.5	268.0	138.11	9.25	147.36	4.834	9.730	4.15	0.251	5.085	40.73	3.552
2..	2.486	0.0108	2.497	208.1	207.65	117.35	5.88	123.23	4.043	8.939	3.51	0.191	4.234	31.56	3.623
3..	1.951	0.0059	1.956	144.0	143.65	95.72	3.40	99.12	3.252	8.148	2.76	0.119	3.371	21.83	3.527
4..	1.471	0.0023	1.473	95.0	94.66	74.04	1.45	75.49	2.477	7.373	1.95	0.062	2.539	14.38	3.554
5..	1.014	0.0008	1.014	54.0	53.70	50.02	0.73	50.75	1.665	6.561	1.24	0.024	1.689	8.16	3.785
				EXPERIMENT No. 6.	MAY 27TH, 1899. P. M.										
1..	......	......	2.806	247.0	246.5	129.61	7.95	137.56	4.513	9.622	3.86	0.232	4.745	37.45	3.623
2..	......	......	2.538	213.0	212.55	116.33	6.15	122.48	4.018	9.127	3.51	0.192	4.210	32.30	3.739
3..	......	......	2.048	154.3	153.90	94.77	3.60	98.37	3.228	8.337	2.71	0.114	3.342	23.38	3.827
4..	......	......	1.494	96.5	96.15	73.14	1.62	74.76	2.453	7.562	1.92	0.058	2.511	14.61	3.672
5..	......	......	1.062	59.0	58.70	50.21	0.80	51.01	1.674	6.783	1.17	0.021	1.695	8.92	4.042
				EXPERIMENT No. 16.	JUNE 5TH, 1899.										
1..	2.4657	0.0106	2.4763	206.00	205.55	120.47	5.70	126.17	4.140	8.708	3.599	0.199	4.339	31.238	3.456
2..	1.9647	0.0057	1.9704	145.40	145.00	98.97	3.34	102.31	3.357	7.925	2.789	0.125	3.406	22.036	3.406
3..	1.4696	0.0026	1.4722	94.50	94.15	76.19	1.57	77.76	2.551	7.119	2.015	0.063	2.614	14.308	3.386
4..	0.9738	0.0007	0.9745	51.80	51.50	51.48	0.67	52.15	1.711	6.279	1.248	0.024	1.735	7.826	3.424
				EXPERIMENT No. 17.	JUNE 5TH, 1899.										
1..	2.4644	0.0106	2.475	205.50	205.05	121.04	5.70	126.74	4.1583	8.726	3.57	0.178	4.336	31.16	3.491
2..	1.9660	0.0057	1.972	146.00	145.60	97.76	3.33	101.09	3.3168	7.885	2.81	0.123	3.440	22.13	3.468
3..	1.4774	0.0026	1.480	95.70	95.35	75.55	1.63	78.18	2.5651	7.133	2.03	0.062	2.627	14.49	3.403
4..	0.9692	0.0007	0.970	51.40	51.00	51.02	0.70	51.72	1.6969	6.265	1.24	0.024	1.721	7.75	3.433
5..	0.5932	......	0.593	24.50	24.25	32.04	0.30	32.34	1.0612	5.629	0.65	0.006	1.067	3.68	3.339
				EXPERIMENT No. 18.	JUNE 6TH, 1899.										
1..	0.7668	0.0004	0.767	36.00	35.73	38.52	Used flush piezometer.		1.2638	5.91	0.92	0.013	1.277	5.43	3.763
2..	2.9440	0.0167	2.961	267.1	266.60	149.35			4.9001	9.55	4.24	0.280	5.180	40.52	3.437
3..	2.4486	0.0104	2.459	203.5	203.05	125.69			4.1239	8.77	3.52	0.193	4 317	30.86	3.440
4..	1.9469	0.0056	1.952	143.5	143.10	100.83			3.3982	7.96	2.73	0.116	3.424	21.75	3.433
5..	1.4479	0.0025	1.450	92.5	92.15	75.48			2.4765	7.13	1.97	0.061	2.537	14.06	3.479
				EXPERIMENT No. 19.	JUNE 7TH, 1899.										
1..	0.4632	......	0.463	17.10	16.85	26.95	Used flush piezometer.		0.8842	6.162	0.41	0.003	0.887	2.56	3.063
2..	0.4741	......	0.474	17.60	17.35	27.04			0.8872	6.165	0.42	0.003	0.890	2.64	3.144
3..	0.9492	0.0007	0.950	49.50	49.21	51.36			1.6851	6.963	1.08	0.018	1.703	7.48	3.366
4..	1.0014	0.0008	1.002	54.00	53.70	53.42			1.7527	7.031	1.16	0.021	1.774	8.16	3.454
5..	1.4725	0.0026	1.475	95.00	94.65	77.02			2.5270	7.805	1.84	0.053	2.580	14.38	3.470
6..	1.9092	0.0053	1.915	139.50	139.11	97.84			3.2101	8.488	2.49	0.097	3.307	21.14	3.515
7..	2.3961	0.0099	2.406	197.00	197.56	119.34			3.9155	9.194	3.25	0.165	4.080	30.03	3.644
8..	2.9112	0.0162	2.927	262.90	262.41	142.13			4.6633	9.941	4.01	0.250	4.913	39.88	3.662

TABLE No. 26.—REDUCTIONS FOR EXPERIMENTS Nos. 20 AND 21, WHERE THE HEAD ON STANDARD WEIR WAS LESS THAN 2 FT. Mr. Rafter.

Experiment.	Period.	Head on standard weir from middle piezometer, in feet.	Total flow over standard weir by Bazin's formula, in cubic feet.	Flow over experimental weir, corrected for leakage.	Observed head on experimental weir, in centimeters.	Observed head on experimental weir, in feet.	$p + h$.	V_o	$\frac{V_o^2}{2g}$	Final corrected head on experimental weir.	Flow over experimental weir per lineal foot.	$C = M\sqrt{2g}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
20..	1	0.99	53.0	52.7	55.21	1.8114	7.07	1.11	0.016	1.827	8.07	3.272
20..	4	1.96	145.2	144.8	105.92	3.4752	8.73	2.44	0.093	3.568	22.18	3.291
20..	5	1.465	94.5	94.15	80.56	2.6432	7.90	1.76	0.048	2.691	14.42	3.267
21..	1	0.598	25.2	24.94	33.12	1.087	6.34	0.59	0.006	1.093	3.82	3.343
21..	2	0.795	38.5	38.22	44.00	1.444	6.70	0.80	0.010	1.454	5.85	3.337
21..	3	1.007	54.5	54.20	55.13	1.809	7.07	1.11	0.016	1.825	8.30	3.290
21..	4	1.25	74.8	74.48	68.17	2.237	7.49	1.97	0.033	2.270	11.41	3.335
21..	5	0.896	45.8	45.51	49.72	1.631	6.89	0.98	0.015	1.646	6.97	3.301
21..	6	0.710	32.5	32.23	39.42	1.293	6.55	0.74	0.009	1.302	4.94	3.322

TABLE No. 27.—COMPUTATIONS FOR PERIODS (2) AND (3), EXPERIMENT No. 20, TAKING DISCHARGE OVER STANDARD WEIR FROM CURVE COMPUTED FROM RESULTS OF EXPERIMENTS PREVIOUSLY REDUCED.

Period.	Observed head on middle piezometer.	Correction for velocity of approach.	Corrected head on standard weir.	Total flow over standard weir, in cubic feet per second.	Flow over experimental weir.	Observed head on experimental weir.	$\frac{V_o^2}{2g}$	Corrected head on experimental weir.	Flow, per foot of length, over experimental weir.	$C = M\sqrt{2g}$.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2......	2.693	0.013	2.706	233.28	232.81	4.672	0.198	4.875	35.68	3.315
3......	2.441	0.011	2.452	201.76	201.32	4.274	0.160	4.434	30.83	3.302

Mr. Rafter.

TABLE No. 28.—CORNELL EXPERIMENTS. SUMMARY OF ELEVATIONS.

Experiment.	Date.	STANDARD WEIR. Mean elevation of crest.	Elevation gauge zero.	Difference, in centimeters.	Piezometer used in reduction of experiments.	EXPERIMENTAL WEIR. Mean elevation of crest.	Elevation gauge zero.	Difference, in centimeters.	p = height of crest above channel bottom.	Piezometer used in reduction of experiments.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	May 20	95.256	97.926	81.38	Up-stream P.	86.640	86.640	......	4.700	Board.
2	" 24	95.258	97.909	80.80	" P.	86.827	86.827	......	4.887	"
3	" 26	95.258	94.262	30.36	" P.	86.847	86.827	......	4.907	"
4	" 26	95.258	94.262	30.39	" P.	86.847	86.827	......	4.907	"
5	" 27	95.258	94.261	30.39	Middle P.	86.836	87.342	15.42	4.896	Up-stream P.*
6	" 27	95.258	94.261	30.39	" P.	87.049	87.342	8.93	5.109	" P.
7	" 29	95.258	94.261	30.39	" P.	86.835	87.342	15.45	4.895	" P.
8	" 30	95.258	94.259	30.45	" P.	86.835	87.349	15.67	4.895	" P.
9	" 31	95.258	94.259	30.45	" P.	86.881	87.349	14.26	4.941	" P.
10	June 1	95.258	94.259	30.45	" P.	86.510	87.349	25.57	4.570	" P.
11	" 1	95.258	94.259	30.45	" P.	86.510	87.349	25.57	4.570	" P.
12	" 2	95.258	94.259	30.45	" P.	86.504	87.349	25.76	4.564	" P.
13	" 2	95.258	94.259	30.45	" P.	86.504	87.349	25.76	4.564	" P.
14	" 3	95.258	94.259	30.45	" P.	86.469	87.349	26.82	4.529	" P.
15	" 3	95.258	94.259	30.45	" P.	86.469	87.349	26.82	4.530	" P.
16	" 5	95.258	94.259	30.45	" P.	86.508	87.349	25.63	4.568	" P.
17	" 5	95.258	94.259	30.45	" P.	86.508	87.349	25.63	4.568	" P.
18	" 6	95.258	94.259	30.45	" P.	86.590	87.349	23.13	4.650	Flush P.
19	" 7	95.258	94.259	30.45	" P.	87.218	87.349	3.99	5.278	" P.
20	" 10	95.258	94.259	30.45	" P.	87.197	......	4.63	5.257	" P.
21	" 12	95.258	94.259	30.45	" P.	87.197	......	8.38	5.257	" P.

Elevation of bottom of channel, back of bulkhead at experimental weir = 81.94 ft.
Height of crest of standard weir above bottom of channel of approach = 13.13 ft.
Length of crest of standard weir.. = 16.00 ft.
Length of crest of experimental weir, Nos. 1-19........................ = 6.58 ft.
Length of crest of experimental weir, Nos. 20-21....................... = 6.53 ft.

* Placed 8 ins. above the bottom.